End of Topic Test – Chapter 3: Modelling and Estimation — Mark Scheme

Total: 47 marks

Note: this test consists mainly of open-ended Fermi-estimation questions. Numerical answers will vary depending on the assumptions made. Award method/accuracy marks for any sensible, clearly stated assumption and consistent, correctly-executed arithmetic, provided the final answer is of a realistic order of magnitude. Example solutions are given below.

1. Fermi estimation: mobile phones

(a) B1+B1 (any two) reasonable assumptions, e.g. assume the average number of phones per attendee (accounting for some people carrying two, e.g. a personal and a work phone, and some carrying none, e.g. young children); assume the full 72,000 attendance figure represents the number of individuals present at any one time. *[2]*

(b) M1 assumption quantified, e.g. average ≈ 1.1 phones per person. M1 $72{,}000 \times 1.1$. A1 estimate $\approx 79{,}200$, i.e. of order 8×10^4 phones (accept any answer in the range roughly 60,000–90,000 with valid supporting method). *[3]*

(c) B1+B1 (any one, developed) limitation, e.g. the assumption of ≈ 1 phone per person may not hold uniformly (families sharing devices, staff carrying multiple phones); the true attendance may fluctuate throughout the event so 72,000 may not all be present simultaneously. *[2]*

2. Standard form and scaling

Speed of light 3.0×10^8 m/s; satellite height 4.2×10^7 m.

(a) M1 time = distance ÷ speed = $\dfrac{4.2 \times 10^7}{3.0 \times 10^8}$. A1 = 0.14 s. *[2]*

(b) M1 recognise time $\propto$ distance (direct proportion, since $t = d/v$ with v fixed), so doubling distance doubles time. M1 new distance = 8.4×10^7 m. A1 new time = 0.28 s. *[3]*

3. Subdividing: estimating streetlights in a city

(a) B1+B1 (any two) assumptions, e.g. streetlights are placed evenly every 30 m with no gaps (e.g. at junctions/roundabouts); lights are on only one side of the road (or a consistent assumption is made about both sides); the dashed grid lines on the map represent all the main roads in the city (no other roads need lighting). B1 a third valid assumption may also be credited if two are not fully developed. *[3]*

(b) M1 total length of roads running one direction: from the diagram, 5 roads each 6 km long = 30 km. M1 total length of roads running the other direction: 3 roads each 10 km long = 30 km (accept alternative sensible readings of the grid with consistent working). M1 total road length = 60 km = 60,000 m. A1 number of streetlights = $60{,}000 \div 30 = 2{,}000$ (accept $\approx 4{,}000$ if candidate consistently assumes lights on both sides of every road, with this stated). *[4]*

4. Modelling cycle: estimating water usage

(a) B1+B1+B1 (any three) reasonable, quantified assumptions, e.g. each student uses the toilet ≈ 3 times per day using ≈ 6 litres per flush; each student drinks ≈ 1 litre of water during the day; each student washes their hands ≈ 5 times using ≈ 0.5 litres each time. *[3]*

(b) M1 toilet use: $1{,}200 \times 3 \times 6 = 21{,}600$ litres. M1 drinking + handwashing: $(1{,}200 \times 1) + (1{,}200 \times 5 \times 0.5) = 1{,}200 + 3{,}000 = 4{,}200$ litres. M1 total = $21{,}600 + 4{,}200$. A1 total $\approx 25{,}800$ litres ($\approx 2.6 \times 10^4$ L), accept any answer of a similar order of magnitude with valid supporting assumptions. *[4]*

(c) B2 (or B1+B1) valid improvement, e.g. use actual water-meter readings from the school for a typical day instead of estimated assumptions; account for variation between days (e.g. PE days involve showers and use more water). *[2]*

5. Estimating the volume of a tree trunk

Diameter ≈ 0.6 m (radius 0.3 m), height ≈ 5 m.

(a) M1 formula $V = \pi r^2 h$. M1 substitute $r = 0.3$, $h = 5$: $V = \pi \times 0.3^2 \times 5$. A1 $V \approx 1.41$ m^3. *[3]*

(b) B2 (or B1+B1) valid limitation, e.g. a real trunk tapers (narrower at the top than the base) so is not a perfect cylinder, meaning the cylindrical model likely overestimates the true volume; bark texture/buttress roots at the base add further irregularity. *[2]*

6. Standard form: density of the Earth

Mass 6.0×10^{24} kg, volume 1.1×10^{21} m^3.

(a) M1 density = mass ÷ volume. M1 $6.0 \div 1.1 \approx 5.45$ and $10^{24} \div 10^{21} = 10^3$ combined correctly. A1 density $\approx 5.5 \times 10^3$ kg/m^3 (accept 5.45×10^3). *[3]*

(b) B2 (or B1+B1) valid reason, e.g. standard form allows very large or very small numbers to be written, read and compared easily without long strings of zeros; it makes calculations (multiplying/dividing powers of ten) quicker and less error-prone. *[2]*

7. Assumptions: estimating the number of fish in a lake

Lake surface area 2.5 km^2.

(a) B1+B1+B1 (any three) assumptions, e.g. an assumed average fish density per km^2 (a stated numerical value); fish are assumed to be evenly distributed across the lake; the lake's depth/volume is assumed roughly uniform if a volume-based method is used; no significant seasonal variation in fish numbers. *[3]*

(b) M1 assumed density stated, e.g. ≈ 800 fish per km^2. M1 correct multiplication: 2.5×800. A1 estimate = 2,000 fish (accept any answer of a similar order of magnitude, e.g. 1,000–5,000, with a clearly stated and multiplied assumption). M1 working clearly and logically laid out (assumption $\rightarrow$ calculation $\rightarrow$ estimate). *[4]*

(c) B2 (or B1+B1) valid improvement, e.g. take an actual sample (netting or electrofishing a small area) to estimate the true density, then scale up to the whole lake; or use a capture–recapture method to estimate the total population directly rather than assuming a density. *[2]*