

End of Topic Test – Chapter 6: Confidence Intervals

Answer all questions. Show all working and give clear interpretations where requested.

1. Confidence interval for a mean

A sample of $n = 25$ patients attending a clinic had systolic blood pressure readings (mmHg) with sample mean $\bar{x} = 132.4$ and sample standard deviation $s = 9.6$.

- (a) (5 points) Calculate a 95% confidence interval for the population mean systolic blood pressure. Use the t -distribution and give your answer to one decimal place. Show all steps, including the critical t -value you use.

- (b) (3 points) Interpret the interval in context. State clearly what the 95% confidence level means.

- (c) (2 points) Explain briefly why the t -distribution is used here rather than the Normal z -distribution.

2. Confidence interval for a proportion

A university surveys $n = 400$ students about whether they support a proposed change to the timetable. 260 students say they support the change.

- (a) (4 points) Calculate a 95% confidence interval for the true proportion who support the change. Give your answer as percentages to one decimal place.
- (b) (3 points) The university claims that at least 60% of students support the change. Using your interval, assess whether this claim is supported by the sample data. Explain your reasoning.
- (c) (2 points) State one condition that should be checked before using the Normal approximation for a proportion confidence interval, and verify whether it holds here.

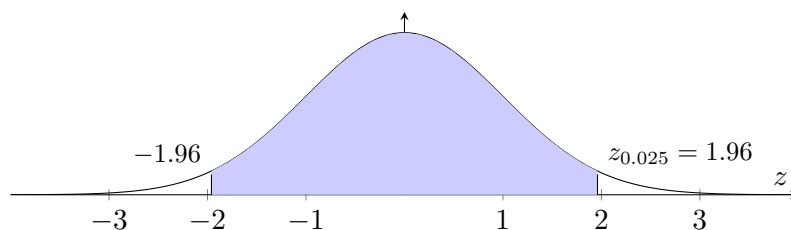
3. Sample size for a desired margin of error

A researcher wants a 95% confidence interval for a population mean to have a margin of error no greater than 0.5 units. From a pilot study the population standard deviation is estimated as $\sigma \approx 3.2$.

- (a) (4 points) Calculate the minimum sample size required. Show your working and give the final n as a whole number.
- (b) (2 points) If the researcher instead wanted a 99% confidence level, explain qualitatively how the required sample size would change and why.

4. Critical values and confidence levels

The diagram below shows a standard Normal curve with the central 95% region shaded.



- (a) (3 points) Explain how the critical value $z_{0.025} = 1.96$ is used when constructing a 95% confidence interval for a mean with known σ .
- (b) (2 points) State the critical value you would use for a 99% confidence interval (Normal approximation) and explain briefly how the interval width would change compared with the 95% interval.

5. Interpretation and misconceptions

Answer the following short questions.

- (a) (3 points) A student calculates a 95% confidence interval for a mean as (10.2, 13.8). Write a correct sentence interpreting this interval for a non-technical audience.

- (b) (3 points) Explain why it is incorrect to say: “There is a 95% probability that the population mean lies between 10.2 and 13.8.”

- (c) (2 points) Give one practical reason why a researcher might prefer a wider confidence interval.

A manufacturer tests a new process to reduce the mean weight of a product. A random sample of $n = 50$ items produced under the new process has mean weight $\bar{x} = 99.1$ grams and sample standard deviation $s = 1.8$ grams. The target mean weight is 100 grams.

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