

Sampling Methods and Random Numbers

Paper 1 or Paper 2 — Common Activity

1 A researcher is investigating buses that arrive in Manchester between 6am and 8am.

- (a) Describe the population in this investigation.
- (b) For her sample, the researcher chooses buses arriving at Piccadilly and Victoria stations.
 - (i) Name the sampling method used.
 - (ii) Give one advantage and one disadvantage of this method.

2 There are 28 students in a class. The teacher wants to select 6 students using random numbers.

Describe clearly how random numbers could be used to make this selection. You should explain what to do if a number is repeated.

3 Opinion polls sometimes fail to predict election results accurately.

Explain what is meant by a **representative sample** and why it is important when carrying out surveys.

4 A sports club has 1.2 million male members and 800 thousand female members. The club wants to send questionnaires to a representative sample of 250 members.

How many females should be included in the sample? Explain your method.

5 A sample is taken from a college sixth form. The head of sixth form chooses a sample stratified by year group and gender.

- (a) Explain why this is a suitable sampling method.
- (b) The table shows the number of students in the sixth form.

	Male	Female
Year 12	140	120
Year 13	110	90

Calculate how many students from each category should be included in a stratified sample of 40 students.

6 Two companies plan to merge.

Company A has 5 000 employees. Company B has 15 000 employees.

A survey receives:

- 500 responses from Company A
- 3 000 responses from Company B

The results are combined and described as a fair representation of both companies.

Does the data support this claim? Show your working.

7 The table shows the number of staff working at a college.

Teaching	Office	Technical	Support
92	48	20	10

The principal chooses 5 people from each category to ask about staff facilities.

- (a) Give two reasons why this is not a good sample.
- (b) Describe a better sampling method.

8 A factory produces light bulbs.

- (a) Describe the population in this context.
- (b) The manager suggests testing every 12th bulb from the production line.
 - (i) Explain why this is not a random sample.
 - (ii) Describe how random numbers could be used to select a random sample.

9

- (a) Explain one similarity between quota sampling and stratified sampling.
- (b) Explain one difference between quota sampling and stratified sampling.

Polls often use online or telephone surveys.

Give reasons why these methods may not produce results that reflect the views of the whole population.

10 A housing association sends questionnaires to all tenants.

The table shows how many questionnaires were sent and returned.

Age	Men		Women	
	Sent	Returned	Sent	Returned
Under 35	240	52	310	61
35–64	620	158	880	182
65+	560	190	600	165

- (a) Comment on whether the responses form a representative sample.
- (b) (i) Calculate the overall response rate.
- (ii) Suggest two ways the response rate could be improved.

11 A regulator monitors customer satisfaction with mobile phone providers.

The table shows market share and the number of customers surveyed.

Provider	Market share	Customers surveyed
Alpha	34%	210
Beta	22%	140
Gamma	18%	95

Comment on whether the survey results fairly represent the market.

Short solutions (step-by-step)

Q1(a)

Step 1: Identify the group being studied.

Answer: All buses arriving in Manchester between 6am and 8am (on the days/time period specified).

Q1(b)(i)–(ii)

Step 1: The sample is taken from convenient locations (two stations).

Answer: Method: opportunity (convenience) sampling.

Step 1: Advantage: quick and easy to collect data.

Step 2: Disadvantage: may be biased and not representative of all buses.

Q2 (random numbers selection method)

Step 1: Label students 01 to 28.

Step 2: Use a random number generator/table to read 2-digit numbers.

Step 3: Ignore numbers outside 01–28.

Step 4: If a number repeats, ignore it and keep going.

Step 5: Stop when 6 different valid numbers are chosen.

Q3 (representative sample)

Step 1: Define “representative”: reflects key characteristics of the population.

Step 2: Explain importance: reduces bias so conclusions/generalisation are valid.

Answer: A representative sample matches the population’s important features (e.g. age, gender). It matters because biased samples give misleading results.

Q4 (female count in representative sample)

Step 1: Total members = 1.2 million + 0.8 million = 2.0 million.

Step 2: Female proportion = $\frac{0.8}{2.0} = 0.4$.

Step 3: Females in sample = $0.4 \times 250 = 100$.

Answer: 100 females.

Q5(b) (stratified sample of 40)

Step 1: Find total sixth form size: $140 + 120 + 110 + 90 = 460$.

Step 2: Compute each stratum: sample = $40 \times \frac{\text{stratum}}{460}$.

Step 3: Round sensibly and adjust so totals sum to 40.

Answer: Y12 M: 12, Y12 F: 10, Y13 M: 10, Y13 F: 8 (one valid rounded allocation).

Q6 (fair representation check)

Step 1: Company A response rate = $\frac{500}{5000} = 0.10 = 10\%$.

Step 2: Company B response rate = $\frac{3000}{15000} = 0.20 = 20\%$.

Step 3: Compare: not proportional (B double A).

Answer: Not fairly representative: Company B is over-represented relative to Company A.

Q7(a) (why not a good sample)

Step 1: Sampling equal numbers from unequal groups distorts proportions.

Step 2: Small groups (e.g. Support) are over-represented; large groups under-represented.

Answer: Two reasons: not proportional; likely unrepresentative/bias towards smaller categories.

Q7(b) (better method)

Step 1: Use stratified sampling by staff category.

Step 2: Choose sample sizes proportional to category sizes.

Step 3: Select individuals within each category using random numbers.

Answer: Stratified random sample proportional to each staff category.

Q8(b)(i) (every 12th bulb not random)

Step 1: Systematic selection can match patterns in production (time-of-day, batches).

Step 2: Not every bulb has equal chance if there is periodic variation.

Answer: Not random because it is systematic and could be biased by production patterns.

Q8(b)(ii) (use random numbers)

Step 1: Number bulbs (or production positions) in a time period 1 to N .

Step 2: Use random numbers to select positions.

Step 3: Ignore repeats/out-of-range; continue until desired sample size reached.

Q9 (quota vs stratified)

Step 1: Similarity: both aim to match proportions of key groups.

Step 2: Difference: stratified selects randomly within groups; quota is often non-random (researcher fills quotas).

Answer: Quota is proportion-based like stratified, but quota usually isn't random whereas stratified is.

Q10(b)(i) (overall response rate)

Step 1: Total sent = $240 + 620 + 560 + 310 + 880 + 600 = 3210$.

Step 2: Total returned = $52 + 158 + 190 + 61 + 182 + 165 = 808$.

Step 3: Response rate = $\frac{808}{3210} \times 100 \approx 25.2\%$.

Answer: About 25.2%.

Q10(b)(ii) (improve response rate)

Step 1: Offer incentives (e.g. prize draw).

Step 2: Send reminders / follow-up emails or letters.

Answer: Any two: reminders, incentives, shorter questionnaire, multiple formats, help completing, deadline extension.

Q11 (comment on market representation)

Step 1: Compare sample proportions with market shares.

Step 2: Compute each provider's sample proportion = $\frac{\text{surveyed}}{\text{total surveyed}}$.

Step 3: If proportions differ notably, sample is not representative.

Answer: Comment should reference proportionality (over-/under-representation) using calculated percentages.

Types of Data and Numerical Measures

Paper 1 or Paper 2 — Common Activity

1 A company records the prices of tablets sold in one week:

£210, £260, £310, £340, £390, £1250, £310

(a) State all the types of data that apply.

– qualitative quantitative discrete continuous primary secondary

(b) Complete the table using a calculator.

Mode	Range	Median	Interquartile Range	Mean	Standard Deviation

(c) (i) Which average best represents the data? Explain your answer.

(ii) Why might the company choose a different average?

(d) Which measure gives the best indication of spread? Explain your answer.

2 Two tutors record the time (in minutes) spent with students.

Time (minutes)	5	6	7	8	9	10	11	12
Tutor A	2	4	7	9	11	10	6	1
Tutor B	5	1	4	8	9	12	7	4

Compare the data for the two tutors. Use an average and a measure of spread.

Box and Whisker Plots

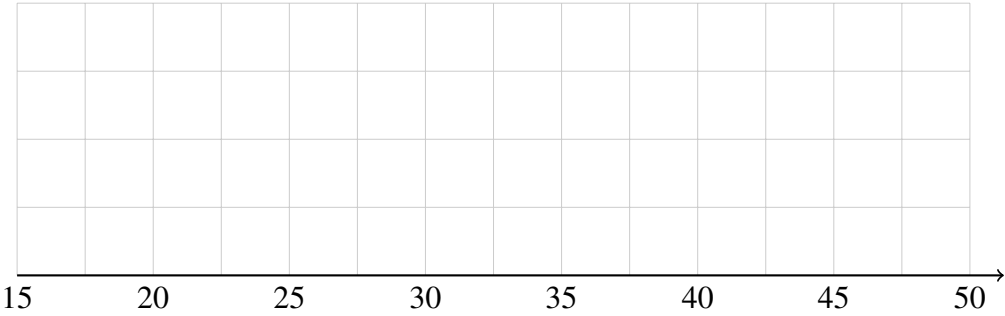
3 The ages of 14 people attending a fitness class are:

18, 22, 25, 27, 30, 32, 35, 37, 39, 42, 45, 48, 51, 55

(a) Complete the table.

Lowest	Lower Quartile	Median	Upper Quartile	Highest

(b) Draw a box plot on the grid below.



(c) Compare the spread and typical age of the group.

Cumulative Frequency Graphs

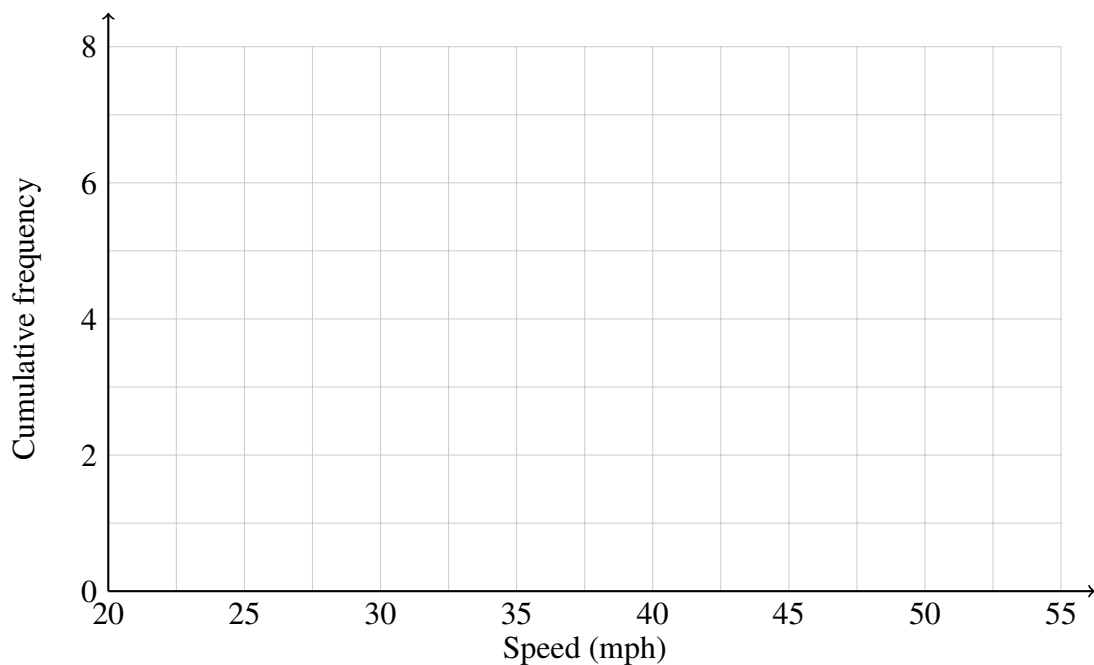
4 A traffic survey records vehicle speeds on a road.

Speed (mph)	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	5	13	21	17	9	3

(a) Draw a cumulative frequency graph on the grid below.

(b) Estimate the median speed.

(c) Comment on whether the speed limit is being followed.



5 A newspaper claims:

“Most drivers exceed the speed limit on this road.”

Using your graph, comment on whether this claim is justified.

Short solutions (step-by-step)

Q1(a)

Step 1: The values are prices (numbers), so they are quantitative.

Step 2: Money is measured on a scale, so treat as continuous.

Step 3: They come from the company's records, so they are secondary data (for you).

Answer: Quantitative, continuous, secondary.

Q1(b)

Step 1: Order the data: 210, 260, 310, 310, 340, 390, 1250.

Step 2: Mode is the most frequent value.

Step 3: Range = max – min.

Step 4: Median is the middle value (4th of 7).

Step 5: Lower half 210, 260, 310 gives $LQ = 260$; upper half 340, 390, 1250 gives $UQ = 390$.

Step 6: $IQR = UQ - LQ$.

Answer: Mode = 310, Range = 1040, Median = 310, IQR = 130. Mean and s.d. from calculator.

Q1(c)

Step 1: There is a large outlier (1250) which pulls the mean upwards.

Step 2: The median is resistant to outliers.

Step 3: A company might prefer the mean if they want a larger “typical” value to report.

Answer: Best representative average: median. Company might use mean to make prices look higher.

Q1(d)

Step 1: Range is affected a lot by the outlier.

Step 2: IQR focuses on the middle 50% and is less affected by extremes.

Answer: Best spread measure here: interquartile range.

Q2

Step 1: Find totals: Tutor A total = $2 + 4 + 7 + 9 + 11 + 10 + 6 + 1 = 50$. Tutor B total = $5 + 1 + 4 + 8 + 9 + 12 + 7 + 4 = 50$.

Step 2: Use a weighted mean: $\bar{x} = \frac{\sum xf}{\sum f}$.

Step 3: Compute $\sum xf$: A = $5(2) + 6(4) + \dots + 12(1) = 445$. B = $5(5) + 6(1) + \dots + 12(4) = 471$.

Step 4: Means: A = $445/50 = 8.9$, B = $471/50 = 9.42$.

Step 5: Use a spread measure (e.g. range or IQR). From the table, both range = $12 - 5 = 7$, but B has more high times so spread is typically larger.

Answer: Tutor B has the higher mean time (≈ 9.42 min vs 8.9 min). Spread: compare using range/IQR from distributions.

Q3(a)

Step 1: The data are already in order.

Step 2: Median is average of 7th and 8th values (since $n = 14$).

Step 3: Lower quartile is median of first 7 values; upper quartile is median of last 7 values.

Answer: Min = 18, $LQ = 27$, Median = 36, $UQ = 45$, Max = 55.

Q3(c)

Step 1: Typical age is the median.

Step 2: Spread is the IQR: $UQ - LQ = 45 - 27 = 18$.

Answer: Typical age = 36 years; middle 50% lie within an 18-year band.

Q4(b) median estimate method

Step 1: Total frequency = $5 + 13 + 21 + 17 + 9 + 3 = 68$.

Step 2: Median position $\approx \frac{68}{2} = 34$ th (or 34.5th).

Step 3: Cumulative frequencies: 5, 18, 39, 56, 65, 68.

Step 4: The 34th value lies in 30–34. Use linear interpolation on the cumulative frequency graph to estimate the median speed.

Answer: Median lies in the 30–34 mph class; estimate it from your graph by reading across from CF = 34.

Q5

Step 1: Use the cumulative frequency curve to estimate $P(\text{speed} > 30)$ or the proportion above 30 mph.

Step 2: If more than 50% are above 30 mph, the headline is supported; otherwise it is not.

Answer: Justify by reading the curve at 30 mph and comparing with total frequency 68.

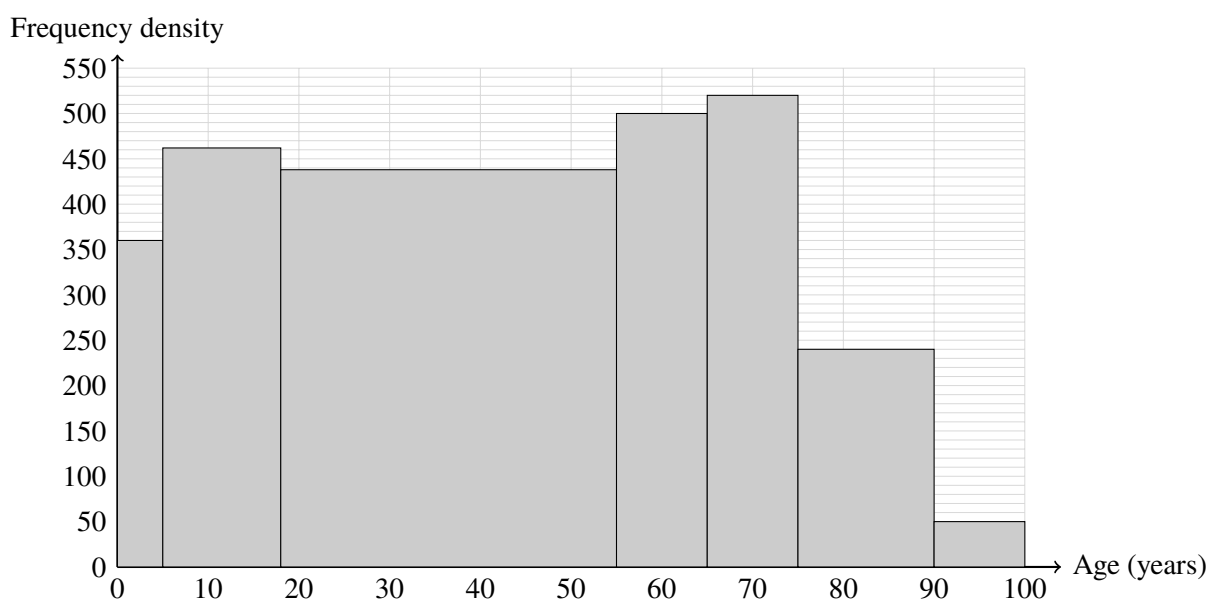
Histograms

Paper 1 or Paper 2 common activity

1 Jack has started a histogram to show the age distribution of the population of a county.

Age (years)	Frequency	Lower boundary	Upper boundary	Class width	Frequency density
0–4		0	5	5	360
5–17	6000	5	18	13	462
18–54	16200		55	37	438
55–64	5000	55		10	500
65–74	5200	65	75		520
75–89	3600	75	90	15	
90+		90	100	10	50

Source: Office for National Statistics (shadow data)



Frequency density is the number of people per year.

Remember that the area of each bar represents the frequency.

(a) Complete Jack's table and histogram. State any assumptions Jack has made.

[4]

(b) What does one small square on the histogram represent? [2]

(c) Estimate:

(i) the number of people aged under 20 [2]

(ii) the percentage of the population aged 60 or over [2]

Borrowing money

Paper 1 or Paper 2 common activity

1 Karl borrows money to buy a new car at an APR of 18%. He has to pay back the loan in two equal repayments of £3800, the first after one year and the second at the end of two years.

Find the amount of the loan. Use the APR formula.

[4]

2 This form shows an application for a loan that has been partially completed.

QuickCash	
How much do you want?	£3200
For how many years?	
Annual repayment	
Total repayment	
APR 42% representative	APPLY

(a) What does the word *representative* mean in this context?

[1]

(b) If the borrower inputs **2** for the length of the loan, what would the annual repayment be?
[2]

(c) Use the APR formula to complete the QuickCash form for a **3-year loan**. [4]

3 The comparative costs of renting and buying depend upon where you live.

City	Monthly rent	Monthly mortgage
Cambridge	£1120	£1850
Newcastle	£610	£520

The cost of a 2-bedroom property.

(a) Suggest why a family in Cambridge might choose the more expensive option of buying.
[2]

(b) Suggest why a family in Newcastle might choose the more expensive option of renting.
[2]

4 Sheila pays a deposit of 25% for a £240 000 flat. The annual interest on her mortgage is 4% and she repays £1000 per month.

This table shows part of a spreadsheet for her repayments.

Year	Initial amount	Interest	Repayments
2017	£180 000	£7 200	£12 000
2018	£175 200		
2019			
2020			

(a) Complete the first three rows of the spreadsheet. [3]

(b) In which year will Sheila pay off the mortgage?

[2]

Budgeting

Paper 1 or Paper 2 common activity

1 Will and Kate are starting to save for a 10% deposit on a house. They are using a savings account at 1.5% AER. They aim to buy a £250 000 house in four years, when Kate has finished her degree. They use an online calculator.

Try our simple savings calculator!			
Amount saved	£4 943.74		
Start date	Jan 2017	End date	Jan 2021
Monthly saving	£100		
Interest rate (AER)	1.5%		

(a) What do the initials AER stand for? [1]

(b) The monthly saving of £100 is not sufficient. What amount of monthly saving do Will and Kate need to make to reach their goal? [3]

2 The account which is offering Will and Kate an AER of 1.5% also publishes a *nominal interest rate* of 1.49%.

Show that this figure is correct and explain why the nominal interest rate is lower than the AER for the same account.

Use the AER formula. [4]

3 Steve deposits £2000 in a Best Saver account. It earns compound interest of 0.99% every 3 months.

(a) How much does Steve have in his account after 18 months? [3]

(b) Find the nominal rate and the AER for this account. [3]

4 The Johnsons have a net income of £2000 per month. This is their typical monthly expenditure.

Housing/household	£440	Miscellaneous	£200
Transport	£280	Eating out	£160
Food	£240	Clothing	£120
Recreation	£240	Alcohol/tobacco	£40

(a) Without changing their expenditure, what percentage of their income could the Johnsons use to build up their savings? [3]

(b) How much will the Johnsons be able to save in their working lives? State any assumptions you have made and comment on the amount that they can save. [4]

Taxation

Paper 1 or Paper 2 common activity

See the Preliminary material for a table of taxation rates.

1 During the tax year 2016–2017, Donald was employed on a salary of £82 000. He had a personal tax allowance of £11 000.

(a) Calculate the annual income tax he pays. Split Donald's salary into the different bands.
[4]

(b) One of Donald's colleagues pays £1000 more than Donald in income tax. How much does she earn? [3]

2 During the tax year 2016–2017, Ivana earned £920 per week.

(a) Calculate the amount of national insurance (NI) she pays each week. [3]

(b) After a pay rise of 10%, find the percentage increase in Ivana's NI payments. [3]

3 During the tax year 2016–2017, Sasha was employed on a salary of £27 600. She had a personal tax allowance of £11 000 and, each month, she paid £50 into a personal pension.

Throughout, give your answers to the nearest penny.

(a) Calculate Sasha's monthly NI payments. [3]

(b) Calculate the monthly income tax that she pays. [3]

(c) Sasha has a large student loan and repays 9% of everything she earns over £21 000. Calculate her monthly repayments. [3]

(d) Copy and complete this payslip for Sasha.

Employee No.	Employee	Date	National Insurance No.		
01234	Sasha	30/08/2016	AB123456C		
Payments	Units	Rate	Amount	Deductions	Amount
Salary	1	☐	☐	PAYE Tax	☐
				PAYE NI	☐
				Personal pension	50.00
				Student loan	☐
				Total deductions	☐
Tax period 5	Total gross pay	☐		Totals year to date	
				Total gross pay TD	☐
				Tax paid TD	☐
				NI TD	☐
				Pension TD	☐
				Student loan TD	☐
Tax code 1100L	Payment method: BACS			Net Pay	☐

(a) Completed table

- 0–4: width 5, density $1800/5 = 360$
- 5–17: width 13, density $6000/13 \approx 462$
- 18–54: width 37, density $16200/37 \approx 438$
- 55–64: width 10, density $5000/10 = 500$
- 65–74: width 10, density $5200/10 = 520$
- 75–89: width 15, density $3600/15 = 240$
- 90+: width 10, density $500/10 = 50$

Assumptions:

- Ages are continuous (e.g. 0–4 means $0 \leq \text{age} < 5$).
- The grouped data represents the population accurately.
- Everyone is counted once.

(b) One small square represents $10 \text{ years} \times 10 \text{ density} = 100 \text{ people}$.

(c)(i) Under 20:

$$1800 + 6000 + 2 \times 438 = 8676$$

(c)(ii) Aged 60+:

$$2500 + 5200 + 3600 + 500 = 11800$$

Total population = 34300.

Percentage:

$$11800/34300 \approx 34\%$$

Loan:

$$L = \frac{3800}{1.18} + \frac{3800}{1.18^2}$$

$$= 3220.34 + 2728.25 = 5948.59$$

$$\boxed{L \approx \text{£}5950}$$

- (a) “Representative” means at least 51% of accepted customers receive this APR or better.
 (b) 1-year repayment:

$$3200(1 + 0.42) = 4544$$

- (c) 2-year APR formula:

$$3200 = \frac{R}{1.42} + \frac{R}{1.42^2}$$

$$R(0.7042 + 0.4959) = 3200$$

$$R = 2666$$

Total repayment = £5332.

- (a) Cambridge: buying builds equity, long-term investment, security.
 (b) Newcastle: renting gives flexibility, avoids repairs, avoids needing a deposit.

2018: Interest = $0.04 \times 175200 = 7008$ End balance = $175200 + 7008 - 12000 = 170208$

2019: Interest = $0.04 \times 170208 = 6808.32$ End balance = $170208 + 6808.32 - 12000 = 165016.32$

- (b) Mortgage paid off after about 25–30 years (approx. early 2040s).

Deposit needed = £25,000.
 £100/month gives £4943.74.

$$\frac{S}{100} = \frac{25000}{4943.74} = 5.056$$

$$S \approx £506$$

Required monthly saving: £505–£510.

$$1.015 = \left(1 + \frac{j}{12}\right)^{12}$$

$$1 + \frac{j}{12} = 1.015^{1/12}$$

$$j = 0.0149 = 1.49\%$$

Nominal < AER because AER includes compounding.

(a)

$$A = 2000(1.0099)^6 \approx 2121.40$$

(b) Nominal = $4 \times 0.99\% = 3.96\%$ AER = $(1.0099)^4 - 1 = 4.01\%$

(a) Total spending = £1720 Savings = £280

$$280/2000 = 14\%$$

(b) 40 years:

$$280 \times 12 \times 40 = 134400$$

Taxable = $82000 - 11000 = 71000$

20% on 32000 = 6400 40% on 39000 = 15600

Total = £22000

(b) Colleague pays £23000: Taxable:

$$6400 + 0.4(T - 32000) = 23000$$

$$T = 73500$$

Gross = £84500.

(a) 12% on £672 = 80.64 2% on £93 = 1.86 Total = £82.50

(b) New pay = £1012 2% on £185 = 3.70 New NI = 84.34 Increase = 1.84 → 2.23%

Employee No.	Employee	Date	National Insurance No.		
01234	Sasha	30/08/2016	AB123456C		
Payments	Units	Rate	Amount	Deductions	Amount
Salary	1	2 300.00	2 300.00	PAYE Tax	266.67
				PAYE NI	195.40
				Personal pension	50.00
				Student loan	49.50
				Total deductions	561.57
Tax period 5	Total gross pay	2 300.00		Totals year to date	
				Total gross pay TD	11 500.00
				Tax paid TD	1 333.35
				NI TD	977.00
				Pension TD	250.00
				Student loan TD	247.50
Tax code 1100L	Payment method: BACS			Net Pay	1 738.43

Shadow Workbook: Student Loans

Each year, 9% of everything you earn over £23 000 is repaid. 25 years after graduation, any remaining loan is wiped out.

1 Omar is 62 and works part-time as a librarian. His annual income is £14 800. He is thinking about studying for a History degree and is considering taking out a student loan.

Give Omar advice on whether he should worry about repaying a student loan. [3]

2 The interest rate on a student loan depends on your income. For incomes up to £23 000, the interest rate is simply the RPI (inflation). For higher incomes there is a sliding scale, rising to RPI + 3% for incomes of £43 000 and over.

1. Explain the statement “and so the amount of the loan just keeps pace with inflation”. [2]

2. The RPI is 2.1%. What is the interest rate for someone with an income of £38 000? [3]

3 At the start of a year, Mateo has a student loan of £51 000. During that year he has an income of £35 000 and the RPI is 2.1%.

3. How much does he repay that year? [2]

4. Find the amount of interest added to his loan that year. [3]

5. Assume that his repayments increase by 2% per year. What is the total amount of the loan that Mateo will repay over 25 years? [4]

- 4** Comment briefly on your answer to Question 3 in terms of the two opening statements. [3]

Mark Scheme: Student Loans

Q1 Omar earns £14,800, which is below the £23,000 repayment threshold. Amount above threshold: $14,800 - 23,000 = 0$. Repayment: $0.09 \times 0 = £0$. He will not repay anything unless his income rises; any remaining loan is wiped after 25 years. (3 marks)

Q2(a) If interest = RPI, the loan grows at the same rate as inflation. Its real value stays the same because prices rise at the same rate. (2 marks)

Q2(b) Income above threshold: $38,000 - 23,000 = 15,000$. Proportion of the £20,000 band: $15,000/20,000 = 0.75$. Extra interest: $0.75 \times 3\% = 2.25\%$. Total interest rate: $2.1\% + 2.25\% = 4.35\%$. (3 marks)

Q3(a) Income above threshold: $35,000 - 23,000 = 12,000$. Repayment: $0.09 \times 12,000 = £1,080$. (2 marks)

Q3(b) Proportion: $12,000/20,000 = 0.6$. Extra interest: $0.6 \times 3\% = 1.8\%$. Total interest rate: $2.1\% + 1.8\% = 3.9\%$. Interest added: $0.039 \times 51,000 = £1,989$. (3 marks)

Q3(c) Repayments form a geometric series: First repayment $R_1 = 1,080$, ratio $r = 1.02$, years $n = 25$.

$$S = 1080 \cdot \frac{1.02^{25} - 1}{0.02} \approx 34,560.$$

Accept answers around £34,500–£35,000. (4 marks)

Q4 Mateo repays around £35,000 compared to a £51,000 loan; the rest is wiped. This supports the idea that only high earners repay fully and that the “debt” is less of a burden for lower earners. (3 marks)

Shadow Workbook: Percentage Change

1. Lina buys reusable drinks bottles for £4 each and sells them outside a concert venue for £P. After the concert, she reduces the price by 30% and still makes 40% profit on the cost.

(a) Show that the reduced price is £5.60.

(b) Explain why $0.70P = 5.60$.

(c) Find the original selling price £P.

2. Lina buys enamel pins for £2.40 each and sells them for £C. After a game, she reduces the price by 10% and still makes 20% profit.

Find the selling price £C.

3. The Patels sold their flat for 30% more than the £18 000 they originally paid. This selling price was 10% less than their asking price.

Find the asking price.

4. A group of workers had a wage rise of 4% in March and a further rise of 6% in September. Their new annual salary was £27 456.

Find the initial annual salary.

5. The exchange rate of US dollars to the pound increased by 8%. It then fell by 25% to 1.20 dollars.

Find the original rate.

6. The price of 1 kWh of electricity fell by 7%. It then increased by 18% to 29.5p.

Find the original price.

7. An imported item includes an import duty of 5%. VAT at 20% of this cost also has to be paid and the total charge is £157.50.

Find the cost of the item excluding duty and VAT.

Mark Scheme: Percentage Change

Q1(a) Reduced price: $4 \times 1.40 = \text{£}5.60$. (1 mark)

Q1(b) A 30% reduction leaves 70% of the price $\rightarrow 0.70P = 5.60$. (1 mark)

Q1(c) $P = 5.60/0.70 = \text{£}8.00$. (1 mark)

Q2 Reduced price: $2.40 \times 1.20 = 2.88$. $0.90C = 2.88 \Rightarrow C = 3.20$. (2 marks)

Q3 Selling price: $18,000 \times 1.30 = 23,400$. Asking price: $23,400/0.90 = 26,000$. (2 marks)

Q4 Combined multiplier: $1.04 \times 1.06 = 1.1024$. Initial salary: $27,456/1.1024 = 24,900$. (2 marks)

Q5 Combined multiplier: $1.08 \times 0.75 = 0.81$. Original rate: $1.20/0.81 = 1.48$. (2 marks)

Q6 Combined multiplier: $0.93 \times 1.18 = 1.0974$. Original price: $29.5/1.0974 = 26.9$ pence. (2 marks)

Q7 Total multiplier: $1.05 \times 1.20 = 1.26$. Base cost: $157.50/1.26 = 125$. (2 marks)

Shadow Workbook: Modelling and Estimation

1. Why is managing stock levels important for businesses that sell physical goods?

2. A company repeatedly sells items until stock reaches a minimum level S . A delivery then raises stock to a maximum level $S + D$. This cycle repeats throughout the year.

(a) According to this model, what are the annual sales?

(b) What simplifying assumption is being made about the rate of sales?

(c) What is the purpose of the buffer stock S ?

3. The annual cost of holding stock includes:

• The buying cost £A per item • The delivery/handling cost £B per delivery • The warehousing cost £C per item per year

List several factors that contribute to warehousing costs.

4. A model for the total annual stock cost is:

$$4DA + 4B + \frac{S+D}{2}C$$

Explain the meaning of each term:

(a) $4DA$

(b) $4B$

(c) $\frac{S+D}{2}C$

5. A warehouse expects annual sales of 1400 units. It keeps a buffer stock of 25 units. The buying cost, delivery cost, and warehousing cost are £90, £55, and £4 respectively.

Calculate the total annual stock cost using the model in Question 4.

6. (a) Complete this list of factors that affect the royalties an author earns from a textbook:

- Let the lifetime of the course be L years
- Let each textbook last t years
- Let the number of students per year be ...
- Let the royalty per book be ...

(b) Construct a mathematical model for the total royalties £R earned over the lifetime of the course.

(c) Critically analyse your model. Identify at least one limitation.

7. How many steps would it take to walk around the Earth?

(a) State your assumptions.

(b) Make your estimate, showing all calculations.

8. How many small plastic building bricks would be needed to construct a full-size garage? State your assumptions clearly.

Shadow Workbook: Critical Analysis

1. A website publishes a chart comparing the “truthfulness” of several public figures, based on its own ratings.

(a) Critically analyse this chart as a method of presenting the data.

(b) What does your answer to part (a) suggest about the website’s reliability?

(c) A friend says “ $60 - 45 = 15$, so Person A told 15 more lies than Person B.” Criticise and correct this statement.

2. A small study claims that eating a particular cereal does not significantly reduce cholesterol. The study involved 18 volunteers, most of whom were already health-conscious.

Comment on the reliability of this study and its findings.

3. A 1980s study found that light alcohol consumption by non-smoking pregnant women did not significantly affect birth weight. Explain why repeating this study today would be problematic.

4. An NHS-funded report states that around half of all clinical trial results are never published. Explain why this undermines confidence in the trials that *are* published.

Mark Scheme: Modelling and Estimation

Q1 Stock control prevents shortages, avoids over-ordering, reduces storage costs, and ensures smooth operations.

Q2(a) Annual sales = total drop in stock per cycle $\times$ number of cycles = $4D$.

Q2(b) Assumes sales occur at a constant rate.

Q2(c) Buffer stock protects against unexpected demand or delivery delays.

Q3 Factors include rent, heating, lighting, insurance, security, staff, equipment depreciation.

Q4(a) $4DA$: cost of buying all items delivered across 4 cycles.

Q4(b) $4B$: four deliveries per year, each costing $\pounds B$.

Q4(c) $\frac{S+D}{2}C$: average stock level $\times$ warehousing cost per item.

Q5 Given: $D = 350$, $S = 25$, $A = 90$, $B = 55$, $C = 4$. Total cost = $4DA + 4B + \frac{S+D}{2}C$. = $4(350)(90) + 4(55) + \frac{375}{2}(4)$. = $126,000 + 220 + 750$. = $\pounds 126,970$.

Q6(a) Factors: number of students, book price, royalty rate, replacement rate.

Q6(b) Example model: Books sold per year = students $\times$ (1/t). Total books sold = $L \times \frac{N}{t}$. Royalties = (royalty per book) $\times$ total books sold.

Q6(c) Limitations: assumes constant student numbers, ignores discounts, ignores digital copies.

Q7 Assume Earth circumference $\approx 40,000$ km. Step length ≈ 0.8 m. Steps = $40,000,000/0.8 = 50,000,000$.

Q8 Estimate volume of garage $\div$ volume of one brick. Assumptions about wall thickness, windows, hollow spaces.

Mark Scheme: Critical Analysis

Q1(a) Diagram may be biased, unclear scale, unclear definitions, selective categories, no sample size.

Q1(b) Suggests the website may be selective or biased in how it presents data.

Q1(c) Percentages cannot be subtracted to compare counts; they are proportions, not frequencies.

Q2 Small sample, not representative, participants already health-conscious, low statistical power.

Q3 Unethical to ask pregnant women to drink alcohol; cannot obtain a representative sample today.

Q4 Publication bias: only favourable results appear, giving a distorted picture of effectiveness.