Chapter 2: Personal Finance — Mark Scheme

Rates used throughout (as supplied on the worksheet): Personal allowance £11 200; income tax 20% (first £33 500 of taxable income), 40% (£33 501–£155 000), 45% (over £155 000); NI (non-contracted out) 0% up to £160/week (£700/month), 12% up to £855/week (£3700/month), 2% above; contracted-out rate 10.6% (up to £805/week, £3500/month); student loan 9% of earnings above £18 500 p.a.

Exercise 2A

1. Taxable income $= 24\,150 - 11\,200 = \pounds 12\,950$. This is entirely within the basic-rate band, so tax $= 20\% \times 12\,950 = \pounds 2\,590$.
2. Taxable income $= 58\,150 - 11\,200 = \pounds 46\,950$. Basic-rate band: $20\% \times 33\,500 = \pounds 6\,700$. Remainder $46\,950 - 33\,500 = \pounds 13\,450$ at 40%: $\pounds 5\,380$. Total tax $= 6\,700 + 5\,380 = \pounds 12\,080$.

Exercise 2B

1. Annual salary $= 4500 \times 12 = \pounds 54\,000$.
   a) Taxable income $= 54\,000 - 9\,500 = \pounds 44\,500$.
   b) Tax: $20\% \times 33\,500 = \pounds 6\,700$, plus $40\% \times (44\,500 - 33\,500) = 40\% \times 11\,000 = \pounds 4\,400$. Total $= \pounds 11\,100$.
2. (Method mark question – read from the described graph shape.)
   a) **A** ($\approx \pounds 8\,320$, i.e. $52 \times \pounds 160$): the point where NI contributions start being paid (end of the NI nil-rate band); before this, no tax or NI is paid. **B** ($\approx \pounds 11\,200$): the personal allowance – the point where income tax starts to be paid (NI is already being paid). **C** ($\approx \pounds 44\,700$, where taxable income first reaches $\pounds 33\,500$): both the higher (40%) tax rate begins *and* NI drops from 12% to 2% at almost the same salary ($52 \times \pounds 855 = \pounds 44\,460$), causing a kink where the combined marginal rate jumps from 32% ($20\% + 12\%$) to 42% ($40\% + 2\%$).
   b) Using the marginal rates: at A, % taken $\approx 0\%$; at B, $\% \approx \dfrac{12\% \times (11\,200 - 8\,320)}{11\,200} \approx 3\%$; at C (salary $\approx \pounds 44\,700$), tax+NI $= 6\,700 + 4\,341.60 = \pounds 11\,041.60$, i.e. $\approx 25\%$ of salary; at D (a further point on the steeper 42%-marginal-rate line, e.g. salary $\approx \pounds 90\,000$), tax+NI $\approx \pounds 24\,820 + \pounds 5\,247.60 = \pounds 30\,067.60$, i.e. $\approx 33\%$ of salary. (Accept any answer using correct thresholds/method with reasonable read-off values.)
3. Weekly NI on £980: $12\% \times (855 - 160) + 2\% \times (980 - 855) = 12\% \times 695 + 2\% \times 125 = 83.40 + 2.50 = \pounds 85.90$.
4. Daniyar (contracted out, £460/week, between £160 and £805): NI $= 10.6\% \times (460 - 160) = 10.6\% \times 300 = \pounds 31.80$.
5. Priya: annual salary £24 150, allowance £11 200. Monthly salary $= \pounds 2\,012.50$.
   a) Monthly NI: $12\% \times (2\,012.50 - 700) = 12\% \times 1\,312.50 = \pounds 157.50$.
   b) Annual tax $= 20\% \times 12\,950 = \pounds 2\,590$ (as in 2A Q1); monthly $= 2\,590/12 = \pounds 215.83$.
   c) Student loan: annual excess over £18 500 is $24\,150 - 18\,500 = \pounds 5\,650$; $9\% \times 5\,650 = \pounds 508.50$ p.a.; monthly $= 508.50/12 = \pounds 42.38$.
   d) Deductions per month: NI £157.50 + tax £215.83 + pension £50.00 + student loan £42.38 $= \pounds 465.71$. Take-home $= 2\,012.50 - 465.71 = \pounds 1\,546.79$.
   e) Completed payslip (period 6, i.e. 6 months into the tax year):

Payments	Amount	Deductions	Amount
Salary	2 012.50	PAYE Tax	215.83
		PAYE NI	157.50
		Personal pension	50.00
		Student loan	42.38
		Total deductions	465.71

Total gross pay (this period): £2 012.50. Net pay: £1 546.79.
Year-to-date (6 periods): Gross pay TD $= 6 \times 2\,012.50 =$ £12 075.00; Tax paid TD $= 2\,590/2 =$ £1 295.00; NI TD $= 6 \times 157.50 =$ £945.00; Pension TD $= 6 \times 50 =$ £300.00; Student loan TD $= 508.50/2 =$ £254.25.

6. Marcus: expected £85 000, 10% pay rise gives new salary $85\,000 \times 1.1 =$ £93 500, an increase of £8 500. Both the old and new salary are comfortably above the NI upper limit (£44 460 p.a.) and above the higher-rate tax threshold, so the entire £8 500 increase falls into the top NI band (2%) and the higher tax band (40%), and is fully above the student-loan threshold.
   a) Extra NI $= 2\% \times 8\,500 =$ £170.00.
   b) Extra tax $= 40\% \times 8\,500 =$ £3 400.00.
   c) Extra student loan $= 9\% \times 8\,500 =$ £765.00. Total extra deductions $= 170 + 3\,400 + 765 =$ £4 335.00. Extra take-home pay $= 8\,500 - 4\,335 =$ £4 165.00.

Exercise 2C

1. a) m = amount of the loan (unknown); $A_1 = A_2 = A_3 =$ £500; $1 + i = 1.18$; $t_1 = 1,\ t_2 = 2,\ t_3 = 3$.
   b) $m = 500(1.18^{-1} + 1.18^{-2} + 1.18^{-3}) = 500(0.84746 + 0.71818 + 0.60863) = 500 \times 2.17427 =$ £1 087.14.
2. Solve $2500 = 1379.46\left[(1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3}\right]$, i.e. the bracket must equal $2500/1379.46 = 1.8123$. Trial gives $i \approx 0.302$ (at $i = 30\%$ the bracket is 1.8161; at $i = 31\%$ it is 1.7909; interpolating gives $i \approx 30.2\%$). So the APR implied by the repayment figures is only $\approx 30\%$, **not** the advertised "32%". The advertisement's rate is therefore not actually the APR – it is just an illustrative figure (hence the asterisk and the small-print range of "4.1% to 1180% APR").
3. a) $800(1+i)^2 = 1100 \Rightarrow (1+i)^2 = 1.375 \Rightarrow 1 + i = 1.1726 \Rightarrow i = 0.1726$, so APR $\approx 17.3\%$.
   b) Check at $i = 13.1\%$: $\dfrac{480}{1.131} + \dfrac{480}{1.131^2} = 424.41 + 375.25 =$ £799.66 $\approx$ £800. This confirms 13.1% is a good approximation to the true APR.
4. a) $1800 = \dfrac{X}{1.24} + \dfrac{X}{1.24^2} = X(0.80645 + 0.65036) = 1.45682X \Rightarrow X =$ £1 235.71.
   b) $1800(1+i)^2 = 2400 \Rightarrow (1+i)^2 = 1.33333 \Rightarrow 1 + i = 1.15470 \Rightarrow$ APR $\approx 15.5\%$.
5. a) Check: $\dfrac{1200}{1.215} + \dfrac{1200}{1.215^2} = 987.65 + 812.94 =$ £1 800.59 $\approx$ £1 800, confirming APR $\approx 21.5\%$.
   b) $1800 = \dfrac{Y}{1.2} + \dfrac{Y}{1.2^2} = Y(0.83333 + 0.69444) = 1.52778Y \Rightarrow Y =$ £1 178.18 per instalment.

Exercise 2D

1. a) Set up columns for n and A_n; put $A_0 = 160\,000$ in the first cell and use the formula `= (previous cell)*1.045 - 13200` filled down for successive years.

b) Continuing the recurrence: $A_4 = 134\,330.85$, $A_5 = 127\,175.74$, $A_6 = 119\,698.65$, $A_7 = 111\,885.09$, $A_8 = 103\,719.92$, $A_9 = 95\,187.32$, $A_{10} = 86\,270.75$, $A_{11} = 76\,952.93$, $A_{12} = 67\,215.81$, $A_{13} = 57\,040.52$, $A_{14} = 46\,407.34$, $A_{15} = 35\,295.67$, $A_{16} = 23\,683.97$, $A_{17} = 11\,549.75$, $A_{18} = -1\,130.51$. Since $A_{17} > 0$ but $A_{18} < 0$, the mortgage is paid off during the **18th year**.

2. Now $A_{n+1} = 1.045A_n - 18\,000$ (annual repayment $= 1500 \times 12$). Continuing: $A_1 = 149\,200$, $A_2 = 137\,914$, $A_3 = 126\,120.13$, $A_4 = 113\,795.54$, $A_5 = 100\,916.34$, $A_6 = 87\,457.58$, $A_7 = 73\,393.17$, $A_8 = 58\,695.86$, $A_9 = 43\,337.17$, $A_{10} = 27\,287.34$, $A_{11} = 10\,515.27$, $A_{12} = -7\,011.54$. So the mortgage is paid off during the **12th year**.

3. Question 1: total repaid $\approx 17 \times 13\,200 + 12\,069.49 \approx$ £236 470 (roughly 18 years' worth of £13 200, minus the fact the last payment is smaller). Question 2: total repaid $\approx 11 \times 18\,000 + 10\,988.46 \approx$ £208 990.

4. a) $1.0055 = 1 + \dfrac{0.55}{100}$ is the monthly growth multiplier: it represents the monthly interest of 0.55% that is added to the outstanding balance each month, before the £1050 repayment is subtracted.

   b) $A_1 = 134\,692.50$, $A_2 = 134\,383.31$, $A_3 = 134\,072.42$, $A_4 = 133\,759.82$, $A_5 = 133\,445.50$, $A_6 = 133\,129.45$.

   c) Amount paid off $= A_0 - A_6 = 135\,000 - 133\,129.45 =$ £1 870.55.

Exercise 2E

1. a) $A = 4256.70 \times 0.0135 =$ £57.47; $B = 4256.70 + 57.47 =$ £4 314.17.

   b) AER $= (1.0135)^2 - 1 = 1.02718 - 1 = 2.72\%$.

2. a) Rate per 3 months $= 2.4\% \div 4 = 0.6\%$.

   b) $A = 905.40 \times 0.006 =$ £5.43; $B = 905.40 + 5.43 =$ £910.83; $C = 910.83$; $D = 910.83 \times 0.006 =$ £5.46; $E = 910.83 + 5.46 =$ £916.29.

   c) AER $= (1.006)^4 - 1 = 1.02422 - 1 = 2.42\%$.

3. a) $4500 \times 1.0205^3 = 4500 \times 1.062769 =$ £4 782.46.

   b) Nominal rate $= 2.05\% \times 2 = 4.10\%$ p.a. AER $= (1.0205)^2 - 1 = 1.04142 - 1 = 4.14\%$.

Exercise 2F

1. Cost $= 312 \div 1.6 =$ £195. Profit $= 312 - 195 =$ £117.

2. A 40% loss means the sale price is 60% of the cost: cost $= 1380 \div 0.6 =$ £2 300.

3. Marked price $=$ cost$\times 2.4$ (140% profit); sale price after 25% reduction $= 2.4 \times 0.75 \times$cost $= 1.8 \times$cost.

   a) Dealer's actual % profit $= (1.8 - 1) \times 100\% = 80\%$.

   b) Cost $= 2160 \div 1.8 =$ £1 200.

4. Pre-VAT price $= 1054 \div 1.175 =$ £897.02. New price at 21% VAT $= 897.02 \times 1.21 =$ £1 085.40. So the correct increase is $1085.40 - 1054.00 =$ £31.40, not £36.89. **The notice is incorrect**: you cannot apply the percentage-point increase in the VAT rate (3.5 percentage points) directly to the VAT-inclusive retail price, because VAT is only charged on the pre-VAT price, not on a price that already includes VAT. The correct new price is £1 085.40 (an increase of about 3.0%), not £1 090.89.

Exercise 2G

1. $215 \div 1.42 = £151.41$.
2. $600 \times 1.18 = €708$; then $708 \times 1.15 = \$814.20$.
3. a) Euros obtained $= 350 \times 1.32 = €462.00$; total cost $= 350 + 12 = £362$.

   b) Commission-free rate needed: $462 \div 362 = €1.28$ per £1.
4. $£1 = \$1.34$ and $£1 = ¥168.50$, so $\$1 = ¥(168.50 \div 1.34) = ¥125.75$.
5. a) Cost in euros $= 310 \times 0.75 = £232.50$. Card fee $= 2.75\% \times 232.50 = £6.39$. Total $= 232.50 + 6.39 = £238.89$.

   b) Rate used $= 254.10 \div 310 = £0.8197$ per euro (about 82p, compared with the true market rate of 75p) – confirming the DCC rate is markedly worse for the customer.

Consolidation Exercise 2

1. Annual salary $= 3950 \times 12 = £47\,400$. Taxable income $= 47\,400 - 9\,800 = £37\,600$.
2. $1000 = V \times 1.04 \Rightarrow V = 1000 \div 1.04 = 961.5\ \text{cm}^3$.
3. a) Total repaid $= 128.40 \times 60 = £7\,704.00$. Interest $= 7\,704.00 - 5\,200 = £2\,504.00$.

   b) Total repaid $= 105.50 \times 60 = £6\,330.00$. Interest $= 6\,330.00 - 5\,250 = £1\,080.00$.

   c) Take the £5 250 loan: despite borrowing £50 more, the total interest is £1 424 lower, so you end up considerably better off overall (this illustrates that a larger loan often carries a much lower APR).
4. a) $550 \times 1.025^4 = 550 \times 1.103813 = £607.10$.

   b) Need $(1 + r)^{12} = 1.025 \Rightarrow 1 + r = 1.025^{1/12} = 1.002060 \Rightarrow r = 0.206\%$ per month; nominal annual rate $= 12r = 2.47\%$.
5. Normal price $= 2\times$cost; sale price for 25% profit $= 1.25\times$cost. Reduction $= \dfrac{2 - 1.25}{2} \times 100\% = 37.5\%$.
6. a) Company A: $350 \times 22.5 = 7\,875$ rand. Company B: $(350 - 6) \times 23.1 = 344 \times 23.1 = 7\,946.4$ rand. **Company B is better** (more rand for the same £350).

   b) Solve $22.5A = (A - 6) \times 23.1 \Rightarrow 22.5A = 23.1A - 138.6 \Rightarrow 0.6A = 138.6 \Rightarrow A = £231$. Below £231, Company A is better; above £231 (e.g. the £350 case above), Company B is better.
7. a) i) $P = 600 \times 1.0015^{12} = 600 \times 1.018150 = £610.89$.

      ii) AER $= 1.018150 - 1 = 1.82\%$.

   b) $P = S \times 1.019^{2n}$.
8. $R = \left(\dfrac{5800}{4500}\right)^{1/7} - 1 = (1.288889)^{1/7} - 1 = 1.03692 - 1 = 3.69\%$.
9. *(Open investigation – answers depend on the household's assumed weekly "basket"; method shown below with one reasonable sample basket: 1 box of eggs, 1 kg onions, 1 kg rice, 2 packets of pasta per week. Award marks for a consistent basket applied correctly across all years.)*

   a) Sample basket per week: 1 eggs(6) + 1 kg onions + 1 kg rice + 2 pasta(500g).

   b) Basket cost (pence): 2016: 224; 2017: 233; 2018: 247; 2019: 261; 2020: 278; 2021: 289; 2022: 307; 2023: 338; 2024: 412; 2025: 449 (i.e. £2.24 rising to £4.49).

c) Annual inflation (%, basket-on-basket): 2016–17: 4.0%; 2017–18: 6.0%; 2018–19: 5.7%; 2019–20: 6.5%; 2020–21: 4.0%; 2021–22: 6.2%; 2022–23: 10.1%; 2023–24: 21.9%; 2024–25: 9.0% – calculated as $\dfrac{\text{new} - \text{old}}{\text{old}} \times 100\%$ each year.

10. a) $95\,000 \times 1.045 - 1750 \times 12 = 99\,275 - 21\,000 = £78\,275$, as required.

    b) Aisha has used the *monthly* repayment (£1750) in a recurrence that steps in *years*; she should subtract the full year's repayments, $12 \times 1750 = £21\,000$, not £1750. Correct recurrence: $A_{n+1} = 1.045A_n - 21\,000$.

    c) i) Formula for cell B6 (i.e. A_4): `=B5*1.045-21000`.

       ii) Continuing: $A_4 = 1.045 \times 42\,533.26 - 21\,000 = £23\,447.26$; $A_5 = 1.045 \times 23\,447.26 - 21\,000 = £3\,502.39$; $A_6 = 1.045 \times 3\,502.39 - 21\,000 = -£17\,340.00$ (negative). Since $A_5 > 0$ but $A_6 < 0$, Aisha pays off the mortgage during **year 6**.

11. Convert all prices to £, using $\$1 = £0.58$ and $\$1 = 1.46$ AUD (so 1 AUD $= £0.58/1.46 = £0.39726$):
    UK: £3.15. USA: $5.25 \times 0.58 = £3.05$. Australia: $6.85 \times 0.39726 = £2.72$.
    The burger was most expensive in the **UK** (£3.15), compared with £3.05 in the USA and £2.72 in Australia.

12. a) $700(1 + i)^2 = 1150 \Rightarrow (1 + i)^2 = 1.642857 \Rightarrow 1 + i = 1.28174 \Rightarrow \text{APR} \approx 28.2\%$.

    b) $700 = \dfrac{X}{1.32} + \dfrac{X}{1.32^2} = X(0.75758 + 0.57392) = 1.33150X \Rightarrow X = £525.75$ per repayment.

    c) Total for lender 1 $= £1\,150$; total for lender 2 $= 2 \times 525.75 = £1\,051.50$. Although the APRs are similar, lender 2's repayment schedule returns some of the capital after just one year, so less interest accrues in the second year; lender 1's method leaves the whole debt (and all its interest) outstanding for the full two years, producing a higher total repayment despite the similar/lower APR.

13. a) $R = (1.021)^{12} - 1 = 1.28327 - 1 = 28.33\%$.

    b) The APR exceeds $12 \times 2.1\% = 25.2\%$ because interest compounds monthly – each month's interest is calculated on a balance that already includes previous months' interest (interest on interest), giving an effective annual rate higher than simply multiplying the monthly rate by 12.

14. a) Second 6 months: interest $= 1766.62 \times 0.0095 = £16.78$, final value $= £1\,783.40$. Third 6 months: interest $= 1783.40 \times 0.0095 = £16.94$, final value $= £1\,800.34$.

    b) Nominal rate $= 0.95\% \times 2 = 1.9\%$ p.a.

    c) AER $= (1.0095)^2 - 1 = 1.01909 - 1 = 1.91\%$.