Chapter 3 Shadow Worksheets

Modelling and Estimation

These shadow worksheets mirror Chapter 3 (Modelling and Estimation) of the AQA Use of Mathematics textbook, exercise by exercise. The numbers, names and contexts have been changed, but each question tests the same skill as the original in the same order, so you can use this worksheet for extra practice or as a parallel assessment.

Reference data and formulae (used throughout this worksheet)

The human body: weight 80 kg; height 1.7 m; lifetime 75 years; heart rate 80 beats per minute; volume of blood 5 litres; sprinting speed 10 m/s; walking speed 3 mph.

The Earth: radius roughly 6000 km; spins on its axis once a day; orbits the Sun once a year.

Data storage: 1 byte = 8 bits; 1 MB = 10^6 bytes.

Equivalences: 1 litre = $1000\,\text{cm}^3$; 1 mile ≈ 1600 m; 1 foot ≈ 30 cm; 1 pound ≈ 500 g; 1 year ≈ 365 days; 1 day = 24 hours; 1 hour = 60 minutes; 1 minute = 60 seconds.

Volume of a cube of side l: l^3. **Surface area of a cube of side l:** $6l^2$.

Volume of a sphere of radius r: $\frac{4}{3}\pi r^3$. **Surface area of a sphere of radius r:** $4\pi r^2$.

Density of water: 1 gram per cubic centimetre.

Exercise 3A

Standard form and Fermi estimation

1. Write these numbers in standard form.

    a) 7000

    b) 0.000004

    c) 62 000 000

    d) 0.000 023

2. Write these correctly in standard form:

    a) 36×10^3

    b) 420×10^5

    c) 0.6×10^3

    d) 0.05×10^5

3. Without using a calculator, work out

    a) $4 \times 10^6 \times 2 \times 10^5$

    b) $(8 \times 10^8) \div (4 \times 10^3)$

    c) $(2.1 \times 10^6) \times (3 \times 10^3)$

    d) $(9 \times 10^6) \div (3 \times 10^4)$

    e) $3 \times 10^{-5} \times 2 \times 10^7$

    f) $(8 \times 10^6) \div (2 \times 10^{-3})$

4. a) By modelling different parts of the body as cuboids (or as cylinders), estimate the volume of your body.

    b) By using the fact that human beings can float in water (but only just!), use your weight to find a really good estimate of your volume.

    When an object floats, the weight of water displaced is equal to the weight of the object. 1 kg of water has volume 1000 cm³.

    c) Compare your estimate from your model in a) with your better estimate in b). How good, or otherwise, was your model in a)?

    d) How could you improve your model in a)?

5. Pascal's constant, k, is a number used in a model of molecular vibration.

$$k = 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,549\,2$$

    a) Express k in standard form.

    b) What approximation to k would you use in a Fermi estimation?

c) Without using a calculator, use your answer to b) to write down an estimate for k^2. (k^2 is a number used in a simplified vibrational energy model.)

6. a) Estimate how long it would take to walk from Cape Town to Cairo.

   b) Carefully list all the simplifications and assumptions that you make.

7. Astronomical distances are expressed in light years. A light year is the distance travelled by light in 1 year.

   a) In a vacuum, light travels 299 792 458 metres every second. Express a light year in metres.

   b) The distance from the Sun to Sirius is 8.6 light years. Express this distance in metres.

   c) New Horizons is a spacecraft that has travelled beyond Pluto. It is travelling at approximately 5.9×10^4 km per hour. Travelling at this speed, how many years would it take to travel between the Sun and Sirius?

8. Blood is roughly 7% of the human body.

   a) Use your estimate for the volume of your body from Q4 to estimate the volume of blood in your body.

   In one minute, all of a person's blood is pumped through their heart.

   b) What volume of blood will your heart pump in a lifetime?

The remaining questions in this exercise are about water usage in one village in Malawi, contrasted with water usage in the UK.

Amina Chirwa lives with her husband and four children in a village in Malawi.

There is no running water in their village. Each day, Amina has to walk to collect water and carry it back for herself and her family.

The nearest source of water is 8 km from her village.

9. Here are some questions you could ask based on this information:

   - How far does she walk each time she goes to collect water?
   - How much water could she carry at a time?

   Write two more questions you could ask.

10. a) List some of the main uses for the water Amina collects.

    b) How much water does Amina need to collect each day for herself and for her family?

11. Estimate how many hours a day Amina spends collecting water.
    What effect do you think this has on her daily life?

12. Roughly 1 in 10 people worldwide do not have access to safe water.

    a) Estimate the number of people who do not have access to safe water.

b) Write a statement comparing your estimate to the number of people in the UK.

13. a) List some of the main uses for water in your home.

    b) How much water does your family use each day?

    c) How much water would a family of four people be likely to use each day in the UK?

> In the UK, we are lucky to have easy access to some of the safest treated water in the world, just by turning on the tap. If you check your findings with figures from the internet, you find estimates of personal (domestic) usage vary a lot.

Activity: Domestic use of water in the UK is relatively small compared to other uses. List some of the major non-domestic uses of water in a developed country like the UK.

Exercise 3B

Estimation techniques: scaling, subdividing and stating assumptions

1. Use a globe or atlas to estimate
   a) the distance from the north to the south of South America
   b) the distance from the west to the east of South America at its widest point
   c) the length of the Kuroshio Current, from Taiwan to Japan.

2. Count your pulse rate for 10 seconds. Use this to estimate how many times your heart will beat from birth until your 21st birthday.

3. Idris used a picture of the globe to estimate the length of Madagascar, from north to south. When he checked his answer he found it to be very inaccurate.
   a) What might he not have noticed, when estimating the length from the globe?
   b) Would this have made his estimate too small or too large?
   c) Suggest how he could improve his method.

4. In the early 20th century, steamships could cross the Atlantic from Southampton to New York in roughly 6 days. Estimate the average speed of these ships in km per hour.

5. In the early 20th century, some shipping lines tended to take noticeably longer than others to cross the Atlantic on the same route, even with similar ships.
   a) What could have affected their journey speeds?
   b) How might some captains have avoided this effect?

Before answering question 6, check your answer to question 5.

6. Make a Fermi estimate of how much longer the slower ships used to take to cross the Atlantic.

Consolidation Exercise 3

The questions in this exercise are grouped by theme.

- Some can be solved in a rough and ready way using Fermi estimation.
- For others you might need to research or collect data.

You can tackle these questions with little technical knowledge as long as you are confident enough to make appropriate simplifying assumptions.

Do questions of both types, on themes that interest you. Make sure you carry out all the stages of the modelling cycle on at least one problem.

- First, use a simple model with quick Fermi estimates of quantities that you need.
- Then evaluate your model by comparing it with the reality of the situation that you are modelling.
- Finally, improve your model by using more accurate data or by developing a more sophisticated model.

Human beings

1. How many times does a person blink in a lifetime?
2. How long will you spend waiting in queues in your lifetime?
3. How far can a person kick a football?
4. How many breaths does a person take in a lifetime?

Meteorology

5. What volume of snow falls on Scotland each year?

Food and drink

6. What weight of bread is consumed in the UK each year?
7. What fraction of an Olympic swimming pool would be filled by the amount of tea or coffee that you will drink in your lifetime?
8. How many cups of tea are drunk each day in the UK?

Science

9. The density of ice is 0.92 grams per cubic centimetre. What is the total mass of the Antarctic ice sheet, given that its volume is estimated at $2.6 \times 10^7\,\text{km}^3$?
10. What is the speed of Nairobi as the city rotates about the Earth's axis?
11. What is the speed of Nairobi in its rotation about the Sun?

Economics

12. What weight of recycling is collected each year by councils throughout the UK?

13. How much will you spend on food in your lifetime?

14. What is the annual cost of electricity used in homes in the UK?

Technology

15. On a Blu-ray disc, the bits of data are stored using a series of tiny pits, more densely packed than on a CD-ROM. Perhaps assume that the pits are arranged in a square array. What is the distance between two adjacent pits?

This rough and ready model will give a sensible answer for the distance between adjacent pits.

Miscellany

16. Throughout the world, how many people are using a mobile phone at any one time?

17. How many roof tiles are there in the UK?

18. What weight of fallen leaves is collected each year from a typical UK garden?

Investigation: The Greenland Ice Sheet

Recent trends in changes to the amount of Greenland ice have led to a number of confusing reports.

> **Greenland ice sheet losing mass at record rate**
> Satellite measurements show that the Greenland ice sheet has been losing ice for over two decades, with the rate of loss accelerating in recent years.

> **Some coastal glaciers in Greenland show seasonal thickening**
> Scientists studying individual glaciers have found that, in winter months, some coastal glaciers temporarily thicken before resuming their long-term retreat.

The two major ice sheets are Greenland and Antarctica. Between them, they contain about 75% of the world's fresh water.

The melting of sea *ice does not greatly affect sea level. Think about a drink with ice cubes in it — as the ice melts, the level of drink in the glass does not change. When land ice (such as the Greenland ice sheet) melts, however, this adds extra water to the oceans and sea level rises.*

While individual glaciers can show short-term local variation, the overall trend for the Greenland ice sheet has been a long-term loss of land ice. This melting of ice is expected to have significant effects, one of which is a rise in sea level.

A detailed survey of Greenland's ice thickness, using satellite radar, has found that the depth of ice varies from 0 to over 3000 metres.

> Greenland is the world's largest island, with an area of roughly $2\,170\,000\,\text{km}^2$.

Estimate the total amount of ice in Greenland, showing full details of all your assumptions and calculations.

Suppose all this ice were to melt. Use your estimate of the amount of ice to calculate what effect this would have on sea level.

Write a report or prepare a presentation on your findings.

Include:

- a critical evaluation of your assumptions and simplifications
- comments on the significance of your conclusions.

Review

After working through this chapter you should:

- be able to write very large and very small numbers in standard form, and calculate with numbers in standard form
- understand what is meant by a Fermi estimate, and be confident making and stating simplifying assumptions

- be able to use scaling and subdividing to break a difficult estimation problem into manageable parts
- be able to use simple formulae (for volume, area, speed, density) to support a model
- be able to critically evaluate a model by comparing it with real data, and suggest improvements.