

Chapter 4 Shadow Worksheets

Critical Analysis

These shadow worksheets mirror Chapter 4 (Critical Analysis) of the AQA Use of Mathematics textbook, section by section. The articles, names and contexts have been changed, but each question tests the same critical-analysis skill as the original in the same order, so you can use this worksheet for extra practice or as a parallel assessment.

Reference: what a well-constructed argument needs (used throughout this worksheet)

Evidence – Is the evidence sound?

Reasoning – Are there holes in the reasoning?

Conclusion – Is the conclusion well-supported?

Clarity may be damaged by: emotive language, vague phrases, unjustified assumptions, self-contradiction.

If two events A and B are correlated, then: this could be coincidental; A could cause B ; B could cause A ; or a third factor could cause both A and B .

4.1 What is critical analysis?

Article A

Concern over the number of junior staff promoted to management here misses the point. No individual talented person working in this company has their professional progress inhibited because of their background.

— HR Director of a national retailer

Question A

What evidence has been used to back up the HR Director's statement? Do you think this is a well-constructed argument?

In March 2016, the Education Secretary said that one in four 16-year-olds left school unable to manage a household budget, and that tackling this would be a priority. Some arguments for and against the ways successive governments have tackled financial literacy were later given in a national newspaper.

As part of a national financial-capability drive launched in November 2015, the Money Advice Service ran a £4 million schools programme to improve budgeting skills. Other initiatives include a £9 million-a-year "Cash Confident" campaign and a £120 million programme for careers guidance in schools.

Article B: Charity accuses government of wasting millions

Dr Okafor took over this month as chair of the National Financial Literacy Trust.

'There have been millions directed to trying to intervene once young people have left school with no money skills, and yet they've all been failures, so we need to tackle causes,' Dr Okafor told the newspaper.

In a stinging assessment of school financial-education programmes, Dr Amara Okafor said that government budgeting campaigns directed at teenagers and young adults 'have all been failures'. Efforts should be devoted to primary-school children and their parents instead, she said.

'Programmes to teach young children about saving and make pocket-money budgeting "normal" should be tested to find ways of stopping poor money habits forming in the first place. I know from my own research that [poor financial habits] actually start in early childhood,' Dr Okafor said.

'A child who never sees their parents budget or save is going to grow up without those habits modelled for them. That child has been set on a trajectory which is leading towards financial difficulty in adulthood. What we don't know yet is how we can intervene.'

Question B

- a) Very briefly, state
 - what evidence Dr Okafor is using
 - what conclusion she draws.
- b) Can you think of some possible flaws in her argument as it is presented here?

The reasoning in many arguments depends on mathematical, especially statistical, ideas. The following extract is typical in this sense but hopefully it is not typical in terms of its flawed logic!

Article C

A blood test provides no evidence of the condition in approximately one out of four patients known to have the condition.

If we take no action on the basis of a negative blood test then there is a 25% chance that we are delaying treatment of a very serious condition.

The author of the article seems to assume that the conclusion follows immediately from the evidence. To check if this reasoning is correct, consider the results for a sample of 1000 patients.

	Condition	No condition
Test positive	12	14
Test negative	4	970

A false positive result is one where the test has incorrectly indicated the presence of the condition. A false negative result is one where the test has incorrectly not indicated the presence of the condition.

Question C

- a) Is the statement in the first sentence of Article C correct? Justify your answer.
- b) Is the statement in the second sentence of Article C correct? Justify your answer.

Key point

An argument will typically consist of a conclusion that is supported by reasoning on the basis of some evidence: **Evidence** (is it sound?) → **Reasoning** (are there holes in it?) → **Conclusion** (is it well-supported?).

4.2 Clarity

If you cannot follow an argument, it may be because it is not expressed clearly enough. Consider, for example, the following extract.

Article D

Diabetic swimmers must be allowed to carry glucose gel during a race, even though it is blatantly obvious that they are gaining an advantage. No sane person could think it sensible to allow one athlete to use a performance aid but then ban other performance aids. However, the disqualification of so many sprinters shows that the ban on altitude tents is fully justified.

The potential overlap of a treatment being needed for medical purposes for some athletes and performance enhancing for others is problematic. You could look up official guidelines and try to do better than the author of Extract D in forming a logical opinion on this tricky issue.

Question D

- a) List examples from this extract of the use of
 - emotive language
 - vague language.
- b) What knowledge does the author, perhaps unjustifiably, assume the reader has?
- c) In what way is the argument given in this extract contradictory?
- d)
 - What assumption does the author appear to be making about glucose gel which is never actually stated?
 - Similarly, what is being implied about the disqualified sprinters?

Now suppose that a friend has returned from a trip to Madagascar and shows you a photograph of an animal you have never seen before, a fossa.

Activity 1

Which of the following deductions, if any, can you make from the evidence of this photograph?

- a) All fossas have reddish-brown fur.
- b) Some fossas have reddish-brown fur.
- c) At least one fossa has reddish-brown fur.
- d) At least one fossa has at least one reddish-brown side.

Key point

When critically analysing an article you must decide if the author is making reasonable assumptions about the subject matter and the reader's knowledge in order to keep the article to an appropriate length. The article may lack clarity if too many assumptions are being made. This may be done deliberately by the author.

Article E

It is damaging to the UK for skilled people to take early retirement, taking their experience and expertise with them. Employers must encourage people to delay retirement beyond pensionable age. In any case, people are living much longer and will need to continue working to avoid poverty in old age.

Question E

- a) What is this extract's conclusion?
- b) What main reason is given? Does it support the conclusion?
- c) Is there anything self-contradictory in this extract?

Article F

It is wrong to throw away any leftover food whilst there are people going hungry in your own town.

Question F

- a) What is the conclusion of this statement and what is given as the reason for this belief?
- b) Does the conclusion follow from the reason?

In the next few activities you can practise applying the ideas you have met so far in this chapter. Article G is quite typical of many letters printed in local newspapers.

Article G

The inappropriate felling of trees in our town centre must be stopped before all our urban greenery disappears. Over 5% of the beautiful tree canopy surrounding our high street has already been removed and so we must act quickly to protect what little remains of the once extensive avenues of mature trees. No reasonable person could therefore support the felling of a large number of healthy oaks for the site of the new car park. This development will make the construction firm rich whilst doing nothing to help the elderly residents of our town who rely on local shops they can walk to.

Question G

Comment fully on this letter. Make sure you consider all the key points of this section: emotive language, vague phrases, unjustified assumptions, self-contradiction.

4.3 Selectivity of data

Coloured dice are sometimes used in tests of so-called “dice influencing”, where a subject tries to mentally influence which face of a six-sided die lands face-up.

There is, of course, a $\frac{1}{6}$ likelihood that a guess will be correct by chance. Despite a mass of negative evidence for such effects, stories such as the following often crop up.

Article H: Mind over dice proved

In extensive tests of 12 000 university students in the UK, one student (who has asked to remain anonymous) correctly predicted eight dice rolls in a row. Dr Voss reported this remarkable performance in a radio interview, saying that the odds were only roughly 1 in 1.7 million that this could have occurred by chance.

Question H

Show that the odds of a person being correct every time when predicting eight dice rolls is indeed roughly 1 in 1.7 million.

Hint: there are 6 possibilities for the first roll, 6×6 possibilities for the first two rolls, and so on.

Although this result sounds remarkable, it is not remarkable at all. In the experiment, each student actually had roughly 1500 separate opportunities to score 8 in a row. So there was a total of $1500 \times 12\,000 = 18$ million opportunities for someone to get 8 in a row.

If this large-scale experiment happened ten times there would be 180 million opportunities to score 8 in a row. As the odds of doing this are 1 in 1.7 million, you would actually *expect* this to happen roughly 100 times across those ten runs!

The following extract about dice influencing is amusing in that following this ‘advice’ would make *not* showing any special skill virtually impossible!

Develop your dice-influencing skills

- The score due to chance is $\frac{1}{6}$ (about 17%). Scores above this show influencing skill. Scores below this show “reverse” influencing skill and these are also important.
- Find the die you get the best results with.
- Find your best time of day for getting good results.
- You should do the test at least 100 times under each set of conditions.

Very much less amusing is the fact that important ideas about statistics can be poorly understood by lawyers and medical experts. As a result, there have been a number of high-profile miscarriages of justice.

You could look up details of real miscarriage-of-justice cases involving statistical evidence.

The precise details of actual cases can be quite complicated but a simple made-up example illustrates the ludicrous nature of some of the arguments used in these cases.

Article I: Cheating raffle winner jailed

Raffle winner Lucky Logan has been jailed for cheating in a charity raffle. It is not known how he carried out this fraud but the statistical evidence was convincing. At his trial, it was pointed out that the probability of his winning by chance was only 1 in 9 million.

Question I

Explain to a non-mathematician what is wrong with the evidence used to convict Logan.

Selectivity of data has naturally occurred whenever you read a newspaper article about someone's research. In the following extract this significantly alters the conclusion.

Article J: Male managers pick male candidates

Only 22% of engineering managers are female and the proportion is even lower in senior roles. Now we know why. Managers in engineering, construction and manufacturing were asked to rate candidates for a post. Each manager got the same application, but some received it from 'David' and others from 'Davina'.

Guess what! Not only were the managers more likely to shortlist David, but any Davina lucky enough to be shortlisted was offered a lower starting salary!

Article K: Engineering managers' subtle gender biases favour male candidates

Despite efforts to recruit and retain more women, a stark gender disparity persists within engineering management. In a randomised double-blind study ($n = 114$), engineering managers from medium and large firms rated the application materials of a candidate – who was randomly assigned either a male or female name – for a junior engineer position. Manager participants rated the male candidate as significantly more competent and hireable than the (identical) female candidate. These participants also selected a higher starting salary and offered more mentoring to the male candidate. The gender of the manager participants did not affect responses, such that female and male managers were equally likely to exhibit bias against the female candidate.

— adapted from a study design used in workplace-bias research

The research paper is the primary source. The newspaper article is the secondary source. Look up the purpose of 'double-blind' studies. The symbol n is typically used to refer to the size of a sample.

Question J & K

- In what significant way has the newspaper article (J) misrepresented the primary data (K)?
- What precisely did the writer of the newspaper article do to cause the misrepresentation?
- Which of the two articles did you find easier to read? List a few of the reasons for this.

Key point

One individual piece of data can seem very convincing but, ideally, you should consider a range of information. In particular, reporting one success is misleading if millions of failures are ignored.

4.4 Sampling and trialling

Whether you have to test the durability of a piece of furniture, the safety of a drug, or the opinions of the electorate, the time and cost involved are good reasons for testing as small a sample as possible.

Key point

When using just a small sample you should take steps to obtain a representative sample so you can draw sensible conclusions. For a good survey you should always include information about the size of the sample and how it was selected.

If a particular group was not represented in the sample, then any conclusions cannot be applied to members of that group.

Activity 2

A local authority commissioned a survey of under-18s in its area in order to plan for future youth services. The sample was obtained by selecting people at several local youth clubs in order that 25% of participants were from each of the age groups 5–8, 9–12, 13–15 and 16–17.

- In what ways could this sample be said to represent the under-18s in this local authority?
- In what ways is the sample likely to be not representative?

The size of the sample you use depends on what you intend to use the results for. Following an opinion poll, support for the Forward Party was reported as:

Forward Party $42.3 \pm 3\%$ ($n = 1640$, 95% confidence level)

A 95% confidence level means that 95 out of 100 polls of this type would give the Forward Party's support at between 39.3 and 45.3%.

Of course, figures such as these refer to a poll on that day. Voting intentions can change significantly in the days before an actual election.

In your everyday life you have to be careful not to slip into the habit of judging things on the basis of very small samples or on the basis of a biased sample.

Article L

Harold ate porridge for breakfast every single day and he lived to be 101.

Question L

What is the significance of this example of long life?

Article M

Mei Lin had not been looking forward to moving when her company relocated her to the Outer Hebrides. Even though she had previously been living on a similarly remote island in Indonesia, she was surprised by the warmth and friendliness of her new island neighbours.

Question M

What sampling error do you imagine Mei Lin might have made which led her to be so pleasantly surprised by the people of the Outer Hebrides?

Trialling is very important for judging things such as the effectiveness of drugs and working practices. However, it is also important that you take various precautions when conducting these trials.

Article N

At one time, it was thought possible that a particular herbal supplement might be useful in preventing motion sickness. One particular trial was conducted by a team of researchers on a long ferry crossing. One third of the passengers in the trial took a standard anti-sickness medication, another third took the herbal supplement, and the remaining third were given pills containing no active ingredients. No-one in the trial knew who was receiving which treatment. For ethical reasons, participants did know what the three possibilities were. Concern amongst passengers about possible side effects of their treatments led to the trial being abandoned partway through the crossing.

Question N

- a) Research the concepts of placebos and control groups. What were the control groups in this trial? What was the purpose of the pills with no active ingredients?
- b) Why was it felt ethical to inform the passengers of the types of treatment?

Key point

Patients can experience beneficial effects simply by believing they are receiving treatment. Control groups taking a placebo can be used to check that the effect of a trialled treatment is greater than this placebo effect.

4.5 Misleading with data

Diagrams are a very effective way to display data and are therefore widely used in marketing and the media. Consider a diagram which dramatically shows the effect of speed on tyre wear, where the vertical axis (tyre wear) is drawn starting at 0.92 instead of 0. Such a diagram would look much less dramatic if the vertical scale were to go from 0 to a sensible maximum.

There is no reason to think that such a diagram was meant to deceive, and even small changes in tyre wear are important. However, the superficial visual effect of a truncated axis would influence many people reading the article accompanying it.

Reporters sometimes deliberately choose the scales and the starting points of axes to mislead and enable headlines to be more dramatic than is justified. The examples in Activity 3 are all based on misuses of diagrams that have actually occurred.

Activity 3

For each of the following diagrams, comment on any unusual features and how it may have been meant to mislead the reader.

- “Drastic effects of new charges”** – A bar chart with only two bars labelled ‘Now’ (28%) and ‘Next year’ (33.8%), with no axis values shown and the y-axis starting partway up.
- “Viewers undecided”** – A box headed ‘Is the new policy fair?’ listing ‘46% Very likely’, ‘29% Likely’, ‘25% Not likely’ with no chart at all, just text.
- “Complaints soar”** – A line graph with only four unevenly-spaced points in time (Mar 09, Sep 09, Jan 11, Aug 12) and unevenly-spaced y-axis labels (10, 13, 18, 24) joined by straight lines.
- “Revenue continues to grow”** – A bar chart of *cumulative* revenue by month, which necessarily rises every month even if monthly sales are falling.
- “Close race for candidates”** – A pie chart with three slices labelled 48%, 55%, and 73%, which sum to far more than 100%.
- “Debate performance affects support”** – A line graph of support over March, April and May with the y-axis inverted (running from 30 at the top down to 0 at the bottom).

Key point

When interpreting a graph, always check carefully that what it appears to be showing is actually the case. The standard conventions are to use uniform scales, starting from 0 on both axes. There are sometimes good reasons for not following these conventions, but this should be clearly flagged to the reader.

Some more complicated examples where you need to criticise data and the way it is used are given below.

Article O: Casualties soar on towpaths

Official statistics show that this year there has been a 40% increase in cycling injuries on towpaths.

Question O

The official statistics were as shown.

Injuries	Towpaths	Roads
2022	5	412
2023	7	458

- Comment on the newspaper article and its headline.
- Why might the number of injuries on towpaths have increased even if each towpath has stayed just as safe?

Article P: Your toddler's babbling may reveal a speech delay

Speech and language delay is a poorly understood condition. There is no single cause, but early intervention can be useful and should be carried out as soon as possible. This intervention can involve extensive practical, emotional and financial commitment for a family.

Researchers at a university speech-and-language lab may have had a break-through in the early diagnosis of speech delay. These findings could enable families to identify concerns in children as young as 12 months old. A small number of 12-month-old babies in the study were later diagnosed with a speech delay. The babbling sounds of these babies were among those which had the lowest variety of syllables measured by the researchers' software.

'We definitely don't want parents to be anxiously analysing their baby's babbling,' said speech therapist Dr Priya Shah. 'The differences in babbling were detected by sophisticated software and not people. It's unclear if the human ear is sensitive enough to detect this.'

Question P

- What criticism could you make of the newspaper's headline?
- What vague words in the newspaper article might make you wonder about the value of the research project?
- In the small study, one baby who was *not* later diagnosed with a speech delay had a babbling sample with a syllable variety which was significantly lower than that of any other baby in the study. What would have been the drawback of early intervention for this baby?
- The only relevant finding of the research paper was that in a sample of 14 babies who babbled during the recording session, the only baby who was later diagnosed with a speech delay had the third lowest syllable-variety score. Comment on the value of the research paper findings and the newspaper article.

4.6 Coincidence?

Making mental connections between things is a natural human activity. Just as babies learn that some actions get them instant attention, adults eventually realised that smoking causes lung cancer. When two things appear to be linked they are said to be **correlated**.

In June 2016, Priya, Marcus and Jana met for the first time on a conference trip in Geneva. After a brief power cut, Priya remarked that she had visited each of the affected regions of Chile, Indonesia, Italy and Turkey shortly before each country experienced a major earthquake in 2010, 2018, 2016 and 2023 respectively. Priya was surprised to discover that Marcus and Jana had also visited all four of the affected regions. Like Priya, Marcus had gone to the regions before the major earthquakes, whereas Jana had always been there shortly after the earthquakes.

Activity 4

Can you think of some reasons why the various correlated events of this story were not as remarkable as they might at first seem?

Hint: think about what careers some of these people may have had which would take them to these places.

Key point

If two events, A and B , are correlated then

- this could be coincidental
- A could cause B
- B could cause A
- a third factor could cause both A and B .

Activity 5

In each of the following scenarios decide whether you think there is a causal link between A and B or whether a third factor might have caused both A and B .

- After Devon moved into their street, the residents noticed an increase in the number of foxes [A] and an increase in the number of bins being raided [B].
- Students who arrive early to a job interview [A] are more likely to be offered the job [B].
- Countries with good university research facilities for engineering [A] have high levels of patent registration [B].
- As sunscreen sales increase [A] so does the rate of melanoma diagnosis [B].
- Rising global internet usage [A] is correlated with rising global obesity levels [B].

4.7 Critical analysis of models

In Chapter 3, you considered and critically evaluated a variety of models from physics, meteorology, medicine and economics. In this section you will analyse models connected with the hotly-debated topic of climate change.

A major source of evidence concerning factors involved in the warming and cooling of the Earth are the records of atmospheric CO₂ and temperature in the Earth's past, taken from ice cores. Over many glacial cycles, troughs correspond to ice ages and peaks to inter-glacial periods such as the present. The two graphs show how closely the changes in atmospheric CO₂ and temperature match each other. With a much finer horizontal scale, it is interesting that for much of the time there is a lag between the two effects, with CO₂ levels following temperature changes by several centuries.

Article S

In a widely-shared online lecture, a commentator stressed the importance of the correlation between the two graphs and concluded:

'There is one relationship which is far more powerful than all the others. When there is more CO₂ in the atmosphere the temperature gets warmer.'

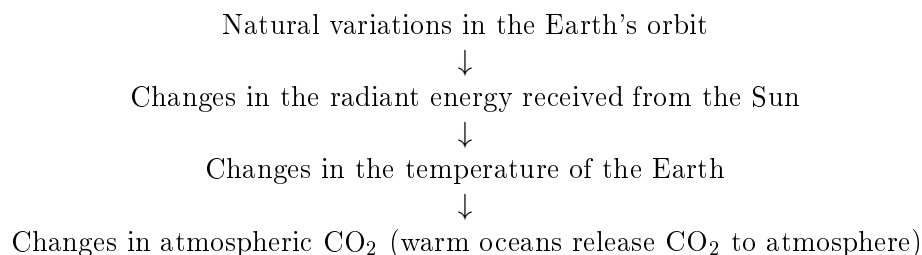
Activity 6

- a) Think of some factors that could affect the amount of energy received from the Sun.
- b) Think of some factors that could affect the amount of energy radiated from the Earth.

Question S

- a) What is the major flaw in the commentator's argument for explaining historic temperature changes in terms of atmospheric CO₂?
- b) Can increases in CO₂ cause climate change?

If you are interested in the historic shifting between warm periods and ice ages, you should research Milankovitch cycles in the Earth's orbit. These are also correlated with variations in temperature and suggest a model along the lines of:



There are perhaps two major issues for you to bear in mind when considering these arguments.

1. People on both sides of the debate can be highly selective about the data they use. The temperature of regions such as the Arctic, central Asia and deep oceans are interrelated in a very complicated way, and in the short term, small changes in one area are probably due to other factors rather than any overall climate change.
2. The temperature of an affected region can fall for long periods whilst the general trend is still upwards.

Key point

Temperature changes (or a lack of them) in a particular region and/or over periods of a few years have no relevance in this debate. Many people (even scientists) are unwilling to suspend judgement. Once you have made your mind up it is often too easy to grasp at supporting evidence and ignore contradictory evidence.

Activity 7: The internet is a rich source of arguments for you to analyse about how much the Earth is warming. Look for a recorded debate or committee hearing on climate policy and identify the main evidence, reasoning and conclusion used by each side.

Consolidation Exercise 4

1. Criticise this visual representation of household energy bills.

Household energy bills triple	
£	£££
2019	2024

2. *It is only fair that drivers should pay higher road tax on larger vehicles. Many studies have shown that larger vehicles cause more road wear on average and so driving a larger vehicle increases damage to infrastructure.*

- a) State the evidence and the conclusion of this extract.
- b) If you assume that larger vehicles do cause more road wear on average, does it follow that 'driving a larger vehicle increases damage to infrastructure'? Carefully justify your answer.

3. In Tomas's year at college there are 280 students. 84 of these students study Further Mathematics. To determine the general attitude of students to the support they receive from tutors, Tomas gives a questionnaire to 18 students in his Further Maths set and to 11 other students. The results are summarised in the table.

	Dissatisfied	Satisfied	Very satisfied	Good	Excellent
Studying Further Maths	2	5	3	6	2
Not studying Further Maths	2	4	2	1	2
Total	4	9	5	7	4

- a) Comment on the categories Tomas has used for this study.
 - b) Tomas reports that 86% of the students in his year are happy with the support they receive. Explain how he arrived at this figure.
 - c) Give two reasons for doubting the figure of 86%.
 - d) Assume that the students in Tomas's set are representative of the students studying Further Mathematics and that the 11 students are representative of the other students. Calculate a better estimate of the percentage of students who are satisfied with the support they receive.
4. Read the following news report.

Hospitals waste £6 billion per year

Hospital trusts have a £30 billion annual procurement budget. However, a report commissioned by the government has found that trusts can save up to 20% on their spending without affecting patient care simply by shopping around. The management consultancy firm they employed looked at spending in two areas.

	Annual spending (£k)	Proposed saving (£k)
Surgical gloves	800	176
Legal services	5200	520

Procurement is obtaining or buying goods or services. £k means thousands of pounds.

- a) Calculate the percentage savings that can be made in each of the two areas of spending.

- b) Describe the evidence and reasoning that might have been used to reach the conclusion of the newspaper headline.
- c) Explain why the newspaper headline is *not* supported by the evidence.
- d) It is suggested that the management consultancy firm should have worked out the saving on the *total* spending in the two areas considered. Carry out this calculation and find how much this would suggest could be saved from the £30 billion budget.
- e) Do you think that your answer to d) is reasonable for the savings that could be made? Justify your answer.

Critical analysis of your own work

The analysis you have learnt to apply when studying the work of other people should be applied to your own work so that any summarising or report writing you do does not contain the many flaws that you have seen in this chapter.

Review

After working through this chapter you should:

- be able to recognise the evidence, reasoning and conclusions of an article
- know that the clarity of an argument can be damaged by emotive language, vague phrases, unjustified assumptions and self-contradiction
- be aware that each side of a debate may be very selective about the evidence they use
- know the importance of studying visual data carefully in case the presentation is misleading
- understand the difference between correlation and causation
- be able to apply your knowledge to the critical analysis of articles and mathematical models.

Investigation

Public health policy

Find a recorded public debate, committee hearing, or panel discussion on a contested public-health or economic policy question (for example, sugar taxes, screen-time limits for children, or minimum alcohol pricing).

Summarise the main speaker's arguments and conclusions, in particular how they answer a clear yes/no question relevant to the policy (for example: 'Has this policy reduced consumption? Yes or No?').

Can you write a brief report putting the speaker's answers in a form that a non-specialist member of the public might have understood?