Chapter 6 Additional Worksheets

Confidence Intervals

These additional worksheets mirror Chapter 6 (Confidence Intervals) of the AQA Use of Mathematics textbook, exercise by exercise: 6.1 / Exercise 6A, 6.2 / Exercise 6B, and 6.3 / Exercise 6C. The numbers and contexts are different from the textbook but test the same skills in the same order, so you can use this worksheet for extra practice or as a parallel assessment.

6.1 Point estimates

Quality control depends on taking a random sample from a population of items and using it to estimate a property – such as the mean – of the whole population.

> **Key point.** The mean of a random sample is called a **point estimate** for the mean of the whole population. A point estimate based on a *larger* sample is generally more reliable than one based on a smaller sample. It is not sensible to draw conclusions from a sample of size 1.

Example (worked, for reference)

A quality inspector weighs the contents of 5 randomly selected bags of flour and obtains 1002 g, 998 g, 995 g, 1004 g and 1001 g.
a) Find a point estimate for the mean weight of the company's bags of flour.
b) A second inspector weighs 40 bags and obtains a point estimate of 999.4 g. Which point estimate is likely to be more accurate? Justify your answer.
Solution.
a) Point estimate $= \dfrac{1002 + 998 + 995 + 1004 + 1001}{5} = 1000\,\text{g}$.
b) The second inspector's estimate (999.4 g) is likely to be more accurate, since it is based on a much larger sample (40 bags rather than 5), so it is less affected by chance variation between individual bags.

Exercise 6A

1. A trading standards officer weighs the contents of 6 randomly selected jars of jam and obtains 452 g, 447 g, 455 g, 449 g, 450 g and 453 g. What point estimate does this give for the mean weight of contents of the company's jars of jam?
2. A second officer weighs a further 30 randomly selected jars and obtains a point estimate of 448.9 g. Which of the two point estimates is likely to be the most accurate? Justify your answer.
3. Use the results of Questions 1 and 2 to obtain a point estimate based on both samples combined.
4. A bottling line is set up so that the volumes of liquid in bottles are normally distributed with mean 500 ml and standard deviation 4 ml. What is the probability that a randomly selected bottle contains less than 495 ml?

6.2 The distribution of the sample mean

If random samples are repeatedly taken from a population, the sample means themselves form a distribution – and this distribution is more tightly clustered around the population mean than individual values are.

Key point. If random samples of size n are taken from a normally distributed population $N(\mu, \sigma^2)$, the sample means $\bar{X}$ have distribution

$$N\left(\mu, \frac{\sigma^2}{n}\right)$$

The standard deviation of this distribution, $\dfrac{\sigma}{\sqrt{n}}$, is called the **standard error of the mean**.

Example (worked, for reference)

The heights of a species of plant are normally distributed with mean 40 cm and standard deviation 6 cm.
a) What is the probability that a randomly chosen plant is taller than 43 cm?
b) What is the probability that the mean height of a random sample of 36 plants is greater than 41 cm?
Solution.
a) $z = \dfrac{43-40}{6} = 0.5$, so $P(X > 43) = 1 - \Phi(0.5) = 1 - 0.6915 = 0.3085$.
b) Standard error $= \dfrac{6}{\sqrt{36}} = 1$. So $\bar{X} \sim N(40, 1^2)$.
$z = \dfrac{41-40}{1} = 1$, so $P(\bar{X} > 41) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$.

Exercise 6B

1. A random sample of size 25 is taken from a normal distribution with mean 50 and standard deviation 10. Find the probability that the sample mean is
   a) greater than 52
   b) less than 47
   c) between 48 and 53.
2. The mass of a species of bird's egg is normally distributed with mean 62 g and standard deviation 5 g. What is the probability that the mean mass of a random sample of 16 eggs is less than 60 g?
3. The number of minutes a commuter's journey takes has an $N(35, 8^2)$ distribution. 100 random samples of 20 journeys are recorded. How many of these samples would you expect to have a sample mean greater than 37 minutes?
4. If a random sample of size 9 is taken from a distribution $N(120, 45)$, find the probability that the sample mean
   a) is greater than 123
   b) lies between 117 and 124.
5. The volumes of a certain type of tree follow a normal distribution with mean $12\,\text{m}^3$ and standard deviation $2\,\text{m}^3$. A forester takes a sample of n trees and finds that the probability the mean volume, V, lies between $11.5\,\text{m}^3$ and $12.5\,\text{m}^3$ is 0.68268. Show that $n = 16$.

6.3 Confidence intervals

A point estimate on its own does not tell you how reliable it is. A **confidence interval** gives a range of plausible values together with a level of confidence.

Key point. If a random sample of size n from a population $N(\mu, \sigma^2)$ has sample mean $\bar{x}$, then a confidence interval for μ has the form

$$\bar{x} \pm z \times \frac{\sigma}{\sqrt{n}}$$

The most commonly used values of z are:

Confidence level	90%	95%	99%
z	1.64	1.96	2.58

The greater the confidence level, the *wider* the interval.

[redacted]

A cereal manufacturer claims that the mean mass of cereal in a box is 750 g with standard deviation 8 g. A trading standards officer selects 16 boxes and finds a sample mean of 745 g. Find a 95% confidence interval for the mean mass of cereal in a box, and comment on the manufacturer's claim.

Solution.

Standard error $= \dfrac{8}{\sqrt{16}} = 2$.

95% CI: $745 \pm 1.96 \times 2 = 745 \pm 3.92$, so the interval is $(741.1, 748.9)$ to 1 d.p.

The claimed mean of 750 g lies **outside** this interval, so on the basis of this sample there is reason to doubt the manufacturer's claim.

Exercise 6C

1. A random sample of 50 lightbulbs from a production line, assumed normally distributed with standard deviation 120 hours, has a sample mean lifetime of 1400 hours. Find a 90% confidence interval for the mean lifetime of all lightbulbs from this line.
2. A random sample of 9 packets of crisps, assumed normally distributed with standard deviation 2.4 g, has a sample mean weight of 34.8 g. Find a 99% confidence interval for the mean weight of a packet of crisps.
3. A manufacturer claims that the mean charge capacity of its batteries is 3000 mAh, with standard deviation 60 mAh. A sample of 25 batteries has mean capacity 2978 mAh. Find a 95% confidence interval for the true mean capacity, and comment on the manufacturer's claim.
4. Explain, without recalculating, whether a 99% confidence interval calculated from the same sample data would be wider or narrower than a 95% confidence interval, and why.
5. A researcher wants a narrower 95% confidence interval for a population mean. State one change to the sampling process that would achieve this, and explain why it works.

Review. After working through this chapter you should:

- understand what a point estimate is and why larger samples give more reliable estimates
- know that sample means from $N(\mu, \sigma^2)$ have distribution $N(\mu, \sigma^2/n)$
- be able to calculate and use the standard error of the mean, $\sigma/\sqrt{n}$
- be able to construct a confidence interval for a population mean at 90%, 95% or 99% confidence
- be able to interpret a confidence interval and comment on claims in context